

# Glossary

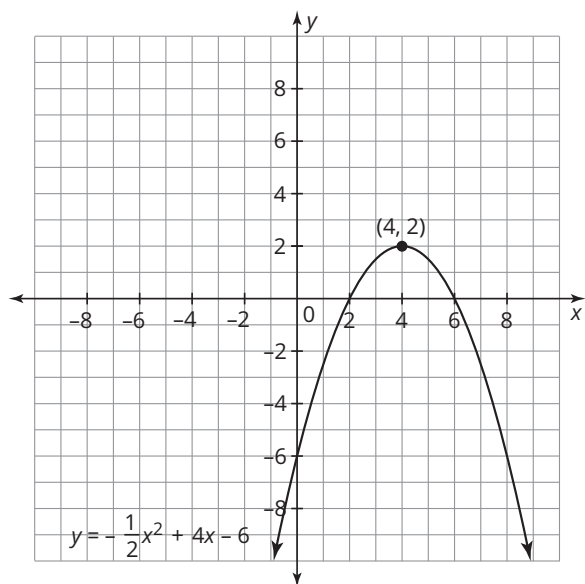
## A

### absolute maximum

A function has an absolute maximum if there is a point that has a  $y$ -coordinate that is greater than the  $y$ -coordinates of every other point on the graph.

#### Example

The ordered pair  $(4, 2)$  is the absolute maximum of the graph of the function  $f(x) = -\frac{1}{2}x^2 + 4x - 6$ .

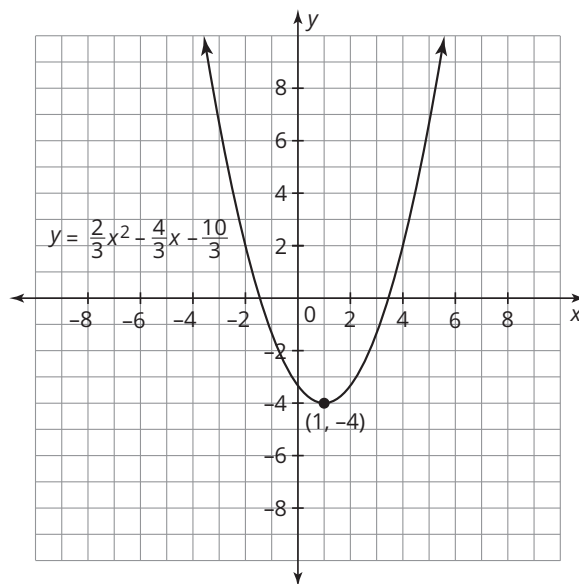


### absolute minimum

A function has an absolute minimum if there is a point that has a  $y$ -coordinate that is less than the  $y$ -coordinates of every other point on the graph.

#### Example

The ordered pair  $(1, -4)$  is the absolute minimum of the graph of the function  $y = \frac{2}{3}x^2 - \frac{4}{3}x - \frac{10}{3}$ .



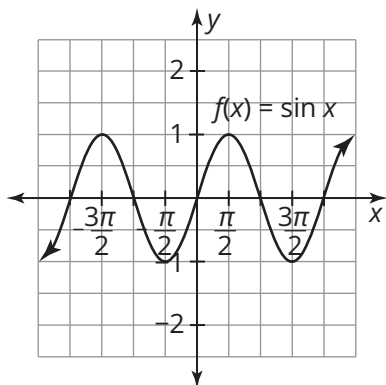
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## amplitude

The amplitude of a periodic function is half of the distance between the maximum and minimum values of the function.

### Example

The function  $y = \sin x$  has a maximum of 1 and a minimum of  $-1$ . The distance between the maximum and minimum is 2. So, the amplitude of  $y = \sin x$  is 1.



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## average rate of change

The average rate of change of a function is the ratio of the independent variable to the dependent variable over a specific interval. The formula for average rate of change is  $\frac{f(b) - f(a)}{b - a}$  for an interval  $(a, b)$ . The expression  $a - b$  represents the change in the input of the function  $f$ . The expression  $f(b) - f(a)$  represents the change in the function  $f$  as the input changes from  $a$  to  $b$ .

### Example

Consider the function  $f(x) = x^2$ .

The average rate of change of the interval  $(1, 3)$  is  $\frac{3^2 - 1^2}{3 - 1} = \frac{9 - 1}{3 - 1} = \frac{8}{2} = 4$ .

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## B

## biased sample

A biased sample is a sample that does not accurately represent all of a population.

### Example

A survey is conducted asking students their favorite class. Only students in the math club are surveyed. The sample of students is a biased sample.

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## Binomial Theorem

The Binomial Theorem states that it is possible to extend any power of  $(a + b)$  into a sum of the form shown.

$$(a + b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n-1} a^1 b^{n-1} + \binom{n}{n} a^0 b^n$$

### Example

Use the Binomial Theorem to find the third term of  $(x + y)^{20}$ .

$$\begin{aligned}(x + y)^{20} &= \binom{20}{2} x^{20-2} y^2 = \frac{20!}{18!2!} x^{18} y^2 \\ &= \frac{20 \cdot 19}{2 \cdot 1} x^{18} y^2 = 190 x^{18} y^2\end{aligned}$$

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## C

## Change of Base Formula

The Change of Base Formula allows you to calculate an exact value for a logarithm by rewriting it in terms of a different base. It is especially helpful when using a calculator.

The Change of Base Formula states:

$$\log_b(c) = \frac{\log_a(c)}{\log_a(b)}, \text{ where } a, b, c > 0 \text{ and } a, b \neq 1.$$

### Example

$$\begin{aligned}\log_4(50) &= \frac{\log 50}{\log 4} \\ &\approx 2.821928095\end{aligned}$$

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## characteristic of interest

A characteristic of interest is the specific question that you are trying to answer or specific information that a study is trying to gather.

### Example

In a sample survey to determine teenagers' online habits, a characteristic of interest is the amount of time that a teenager spends online per day.

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## closed under an operation

A set is closed under an operation if the operation is performed on any of the numbers in the set and the result is a number that is also in the same set.

### Example

The set of whole numbers is closed under addition. The sum of any two whole numbers is always another whole number.

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## cluster

A cluster is a sample of the population that contains the characteristics of the population.

### Example

A city manager randomly selects one block in the city and surveys all of the residents of that block. Each block is considered a cluster.

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## cluster sample

A cluster sample is a sample obtained by creating clusters, with each cluster containing the characteristics of the population, and randomly selecting a cluster.

### Example

If students in a high school are divided into clusters of 20 students based on their student I.D. number and then one cluster is randomly selected, this is a cluster sample.

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## coefficient of determination

The coefficient of determination ( $R^2$ ) measures the “strength” of the relationship between the original data and its regression equation. The value of the coefficient of determination ranges from 0 to 1 with a value of 1 indicating a perfect fit between the regression equation and the original data.

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## common logarithm

A common logarithm is a logarithm with a base of 10. Common logarithms are usually written without a base.

### Example

$\log(10x)$  or  $\log x$  are examples of a common logarithm.

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## composition of functions

Composition of functions is the process of substituting one function for the variable in another function.

### Example

If  $f(x) = 3x - 5$  and  $g(x) = x^2$ , then the composition of the functions  $f(g(x))$  can be written as  $f(g(x)) = 3(x^2) - 5 = 3x^2 - 5$ .

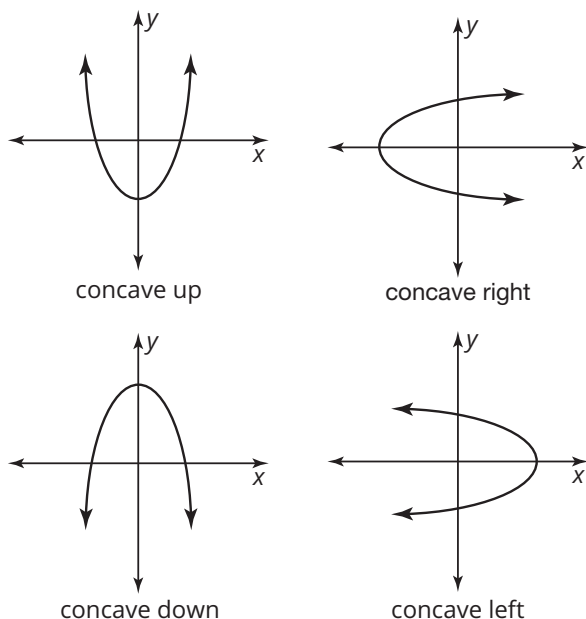
The composition of functions  $g(f(x))$  can be written as  $g(f(x)) = (3x - 5)^2$ .

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## concavity of a parabola

The concavity of a parabola describes the orientation of the curvature of the parabola.

### Example



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## confidence interval

A confidence interval is an estimated range of values, based on the results of a sample survey, that will likely include the population proportion. Typically, a confidence interval of 95%, or 2 standard deviations from the mean, is used. The formula for calculating the confidence interval for proportions is  $\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$ , where  $\hat{p}$  is the sample population and  $n$  is the sample size. The formula  $\frac{s}{\sqrt{n}}$ , where  $s$  is the standard deviation of the sample and  $n$  is the sample size, is used for continuous data.

### Example

A survey of 2000 teenagers reports that 42% have a part-time job.

$$\begin{aligned} &\sqrt{\frac{0.42(1 - 0.42)}{2000}} \\ &\sqrt{\frac{0.42(0.58)}{2000}} \\ &\approx 0.011 \end{aligned}$$

The interval from 40.9% to 43.1% represents a 95% confidence interval for the population proportion.

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## confounding

Confounding is the process of overlooking factors and situations that distort the final results when seeking to gather information or data.

### Example

Suppose that a study is conducted to determine if there is a link between a certain type of insulin that some diabetic patients use and cancer. Confounding can occur due to the fact that there are other potential causes of cancer that could be involved in the sample.

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## continuous data

Continuous data are data that have an infinite number of possible values.

### Example

The heights of students is an example of continuous data.

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## convenience sample

A convenience sample is a sample whose data are based on what is convenient for the person choosing the sample.

### Example

If you choose the students sitting closest to you in math class as your sample, you have a convenience sample.

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## cosine function

The cosine function is a periodic function. It takes angle measures ( $\theta$  values) as inputs and then outputs real number values which correspond to the coordinates of points on the unit circle.

### Example

The function  $h(\theta) = 4 \cos(\theta + \pi)$  is a cosine function.

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## cube root function

The cube root function is the inverse of the power function  $f(x) = x^3$ .

### Example

The cube root function is  $g(x) = \sqrt[3]{x}$ .

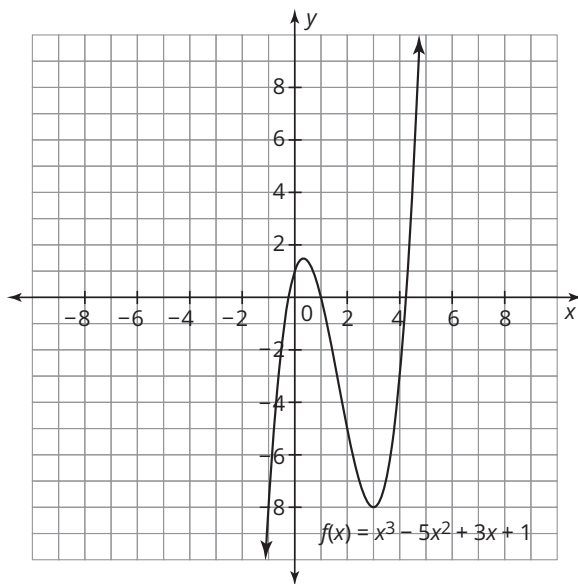
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## cubic function

A cubic function is a function that can be written in the standard form  $f(x) = ax^3 + bx^2 + cx + d$  where  $a \neq 0$ .

### Example

The function  $f(x) = x^3 - 5x^2 + 3x + 1$  is a cubic function.



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## disc

A disc is the set of all points on the circle and in the interior of the circle.

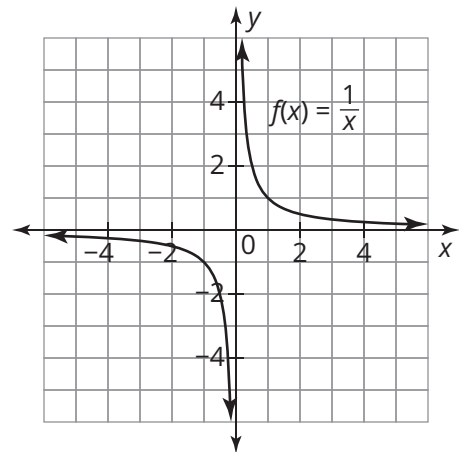
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## discontinuous function

A discontinuous function is a function that is not a continuous curve—it has points that are isolated from each other.

### Example

The function  $f(x) = \frac{1}{x}$  is a discontinuous function.



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## discrete data

Discrete data are data that have a finite number of possible values.

### Example

If you roll a number cube 10 times and record the results, the results are discrete data.

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## D

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## damping function

A damping function is a function that is multiplied to a periodic function to decrease its amplitude over time. It can be from a multitude of function families, including linear, quadratic, or exponential.

### Example

In the function  $f(x) = 2^x \cdot \sin x + 1$ , the exponential function  $2^x$  is the damping function.

## Empirical Rule for Normal Distributions

The Empirical Rule for Normal Distributions states that:

- Approximately 68% of the area under the normal curve is within one standard deviation of the mean.
- Approximately 95% of the area under the normal curve is within two standard deviations of the mean.
- Approximately 99.7% of the area under the normal curve is within three standard deviations of the mean.

### Example

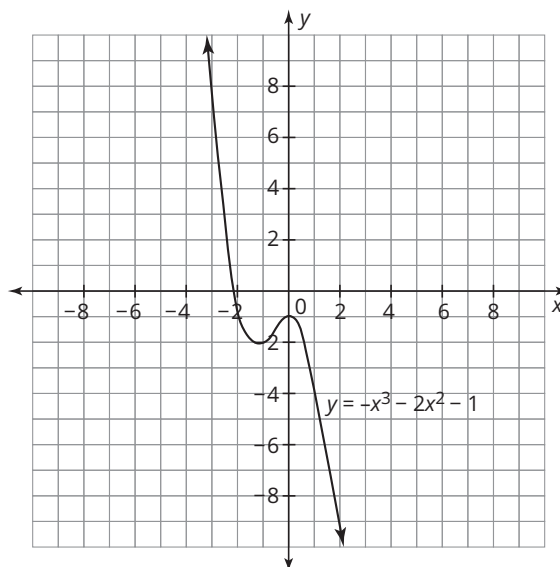
For a data set that is normally distributed with a mean of 10 and a standard deviation of 1, the following are true:

- Approximately 68% of the data values are between 9 and 11.
- Approximately 95% of the data values are between 8 and 12.
- Approximately 99.7% of the data values are between 7 and 13.

## end behavior

The end behavior of the graph of a function is the behavior of the graph as  $x$  approaches infinity and as  $x$  approaches negative infinity.

### Example



The end behavior of the graph shown can be described as follows:

As  $x$  approaches infinity,  $y$  approaches negative infinity.

As  $x$  approaches negative infinity,  $y$  approaches infinity.

## Euclid's Formula

Euclid's Formula is a formula used to generate Pythagorean triples given any two positive integers. Given positive integers  $r$  and  $s$ , where  $r > s$ , Euclid's Formula is  $(r^2 + s^2)^2 = (r^2 - s^2)^2 + (2rs)^2$ .

### Example

Let  $r = 3$  and  $s = 1$ .

$$(3^2 + 1^2)^2 = (3^2 - 1^2)^2 + (2 \cdot 3 \cdot 1)^2$$

$$10^2 = 8^2 + 6^2$$

So, one Pythagorean triple is 6, 8, 10.

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## even function

An even function  $f$  is a function for which  $f(-x) = f(x)$  for all values of  $x$  in the domain.

### Example

The function  $f(x) = x^2$  is an even function because  $(-x)^2 = x^2$ .

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## experiment

An experiment gathers data on the effect of one or more treatments, or experimental conditions, on the characteristic of interest.

### Example

The following is an example of an experiment.

A sample of 200 asthma patients participated in the clinical trial for a new asthma drug. One hundred of the patients received a placebo treatment along with an inhaler, while the remaining 100 patients received the new drug along with an inhaler. Monthly blood and breathing tests were performed on all 200 patients to determine if the new drug was effective.

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## experimental unit

An experimental unit is a member of a sample in an experiment.

### Example

Suppose that an experiment is conducted to test the effects of a new drug on a sample of patients. Each patient is an experimental unit in the experiment.

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## extraneous solution

Extraneous solutions are solutions that result from the process of solving an equation; but are not valid solutions to the equation.

### Example

$$\log_2(x) + \log_2(x + 7) = 3$$

$$\log_2(x^2 + 7x) = 3$$

$$x^2 + 7x = 2^3$$

$$x^2 + 7x = 8$$

$$x^2 + 7x - 8 = 0$$

$$(x + 8)(x - 1) = 0$$

$$x + 8 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = -8$$

$$x = 1$$

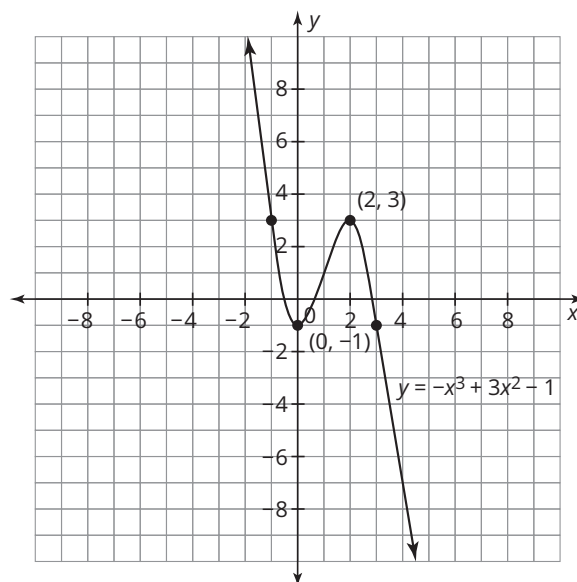
The solution  $x = -8$  is an extraneous solution because the argument of a logarithm must be greater than zero.

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## extrema

Extrema are the set of all relative maximums, relative minimums, absolute maximums, and absolute minimums for a graph.

### Example



The graph shown has 2 extrema, a relative maximum at  $(2, 3)$  and a relative minimum at  $(0, -1)$ .

## Factor Theorem

The Factor Theorem states that a polynomial is divisible by  $(x - r)$  if the value of the polynomial at  $r$  is zero.

### Example

The polynomial  $x^3 - 2x^2 + 2x - 1$  is divisible by  $x - 1$  because  $(1)^3 - 2(1)^2 + 2(1) - 1 = 0$ .

## factored form of a quadratic function

A quadratic function written in factored form is in the form  $f(x) = a(x - r_1)(x - r_2)$ , where  $a \neq 0$ .

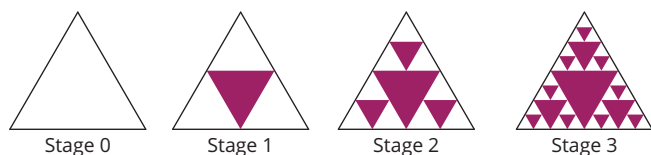
### Example

The function  $h(x) = x^2 - 8x + 12$  written in factored form is  $h(x) = (x - 6)(x - 2)$ .

## fractal

A fractal is a complex geometric shape that is constructed by a mathematical pattern. Fractals are infinite and self-similar.

### Example



## frequency

The frequency of a periodic function is the reciprocal of the period and specifies the number of repetitions of the graph of a periodic function per unit. It is calculated by the formula  $\frac{|B|}{2\pi}$ .

### Example

The function  $f(x) = 3 \cos(2x)$  has a  $B$ -value of 2, so the frequency is  $\frac{|2|}{2\pi}$  or  $\frac{1}{\pi}$  units.

## function

A function is a relation such that for each element of the domain there exists exactly one element in the range.

### Example

The equation  $y = 2x$  is a function. Every  $x$ -value has exactly one corresponding  $y$ -value.

## function notation

Function notation is a way of representing functions algebraically. The function  $f(x)$  is read as “ $f$  of  $x$ ” and indicates that  $x$  is the input and  $f(x)$  is the output.

### Example

The function  $f(x) = 0.75x$  is written using function notation.

## Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra states that any polynomial equation of degree  $n$  must have exactly  $n$  complex roots or solutions; also, every polynomial function of degree  $n$  must have exactly  $n$  complex zeros. However, any root or zero may be a multiple root or zero.

### Example

The polynomial equation  $x^5 + x^2 - 6 = 0$  has 5 complex roots because the polynomial  $x^5 + x^2 - 6$  has a degree of 5.



## geometric series

A geometric series is the sum of the terms of a geometric sequence.

### Example

The geometric series corresponding to the geometric sequence 2, 4, 8, 16 is  $2 + 4 + 8 + 16$ , or 30.

## half-life

A half-life is the amount of time it takes a substance to decay to half of its original amount.

### Example

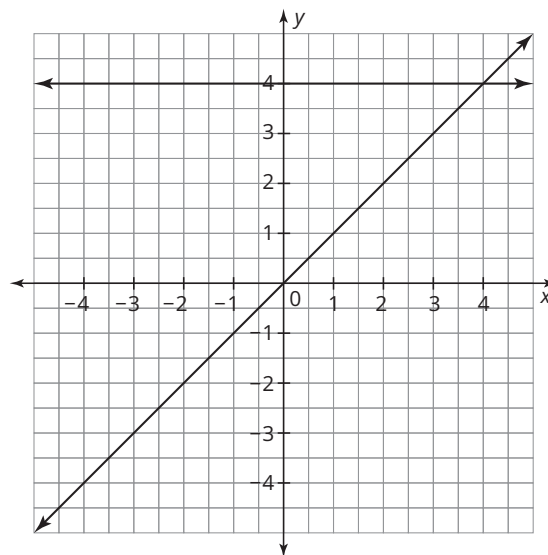
The radioactive isotope strontium-90 has a half-life of about 30 years. A 1000-gram sample of strontium-90 will decay to 500 grams in 30 years.

## Horizontal Line Test

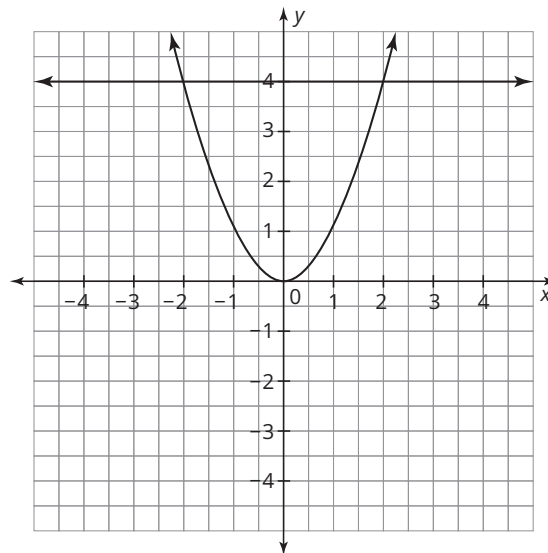
The Horizontal Line Test is a test to determine if a function is one to one. To use the test, imagine drawing every possible horizontal line on the coordinate plane. If no horizontal line intersects the graph of a function at more than one point, then the function is one to one.

### Example

The function  $y = x$  passes the Horizontal Line Test because no horizontal line can be drawn that intersects the graph at more than one point. So, the function is one to one.



The function  $y = x^2$  does not pass the Horizontal Line Test because a horizontal line can be drawn that intersects the graph at more than one point. So, the function is not one to one.



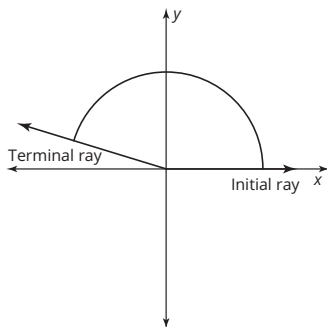
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## initial ray of an angle

The initial ray of an angle in standard position is the ray with its endpoint at the origin and extending along the positive  $x$ -axis.

### Example

The initial ray of the angle is labeled in the diagram.



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## inverse cosine ( $\cos^{-1}$ )

The  $\cos^{-1}$  function is the inverse of the cosine function. The inverse cosine function is written as  $\arccos$  or  $\cos^{-1}$ .

### Example

$$\cos(60^\circ) = \frac{1}{2} \text{ so } \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

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## inverse of a function

The inverse of a one-to-one function is a function that results from exchanging the independent and dependent variables. A function  $f(x)$  with coordinates  $(x, f(x))$  will have an inverse with coordinates  $(f(x), x)$ .

### Example

The inverse of the function  $y = 2x$  can be found by exchanging the variables  $x$  and  $y$ .

The inverse of  $y = 2x$  is  $x = 2y$ .

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## inverse sine ( $\sin^{-1}$ )

The  $\sin^{-1}$  function is the inverse of the sine function. The inverse sine function is written as  $\arcsin$  or  $\sin^{-1}$ .

### Example

$$\sin(30^\circ) = \frac{1}{2} \text{ so } \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

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## inverse tangent ( $\tan^{-1}$ )

The  $\tan^{-1}$  function is the inverse of the tangent function. The inverse tangent function is written as  $\arctan$  or  $\tan^{-1}$ .

### Example

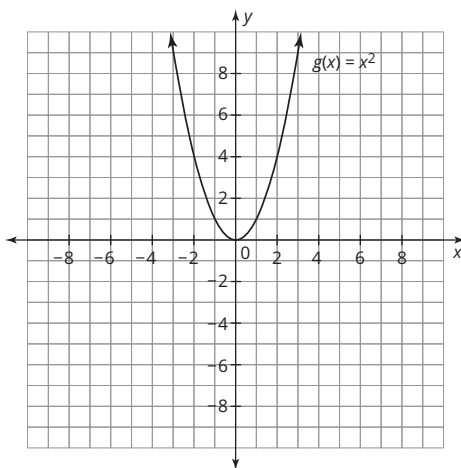
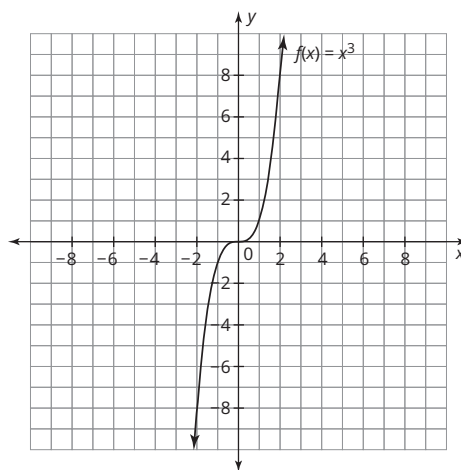
$$\tan(45^\circ) = 1 \text{ so } \tan^{-1}(1) = 45^\circ$$

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## invertible function

An invertible function is a function whose inverse exists. It is one-to-one and passes the Horizontal Line Test, so its inverse will also be a function.

### Example



The graph of  $f(x) = x^3$  is an invertible function because it is one-to-one and passes the Horizontal Line Test. Therefore its inverse will also be a function.

The graph of  $g(x) = x^2$  is not an invertible function because it does not pass the Horizontal Line Test. Its inverse does not exist.

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## iterative process

An iterative process is one in which the output from one iteration is used as the input for the next iteration.

### Example

A recursive sequence uses an iterative process to generate its terms.

$$a_n = 3a_{n-1} + 1$$

$$a_1 = 2$$

Begin with the first term, which is 2, and substitute it into the sequence to get the next term.

$$a_2 = 3a_1 + 1$$

$$= 3(2) + 1$$

$$= 7$$

Then substitute  $a_2$  into the sequence to produce  $a_3$ , and so on.

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## L

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## Law of Cosines

Law of Cosines, or

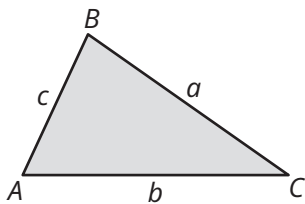
$$a^2 = c^2 + b^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

can be used to determine the unknown lengths of sides or the unknown measures of angles in *any* triangle.

### Example



In  $\triangle ABC$ , the measure of  $\angle A$  is  $65^\circ$ , the length of side  $b$  is 4.4301 feet, and the length of side  $c$  is 7.6063 feet. Use the Law of Cosines to calculate the length of side  $a$ .

$$a^2 = 4.4301^2 + 7.6063^2 - 2(4.4301)(7.6063) \cos 65^\circ$$

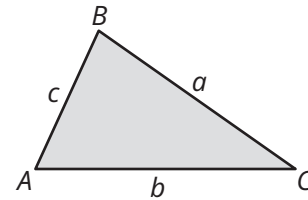
The length of side  $a$  is 7 feet.

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## Law of Sines

The Law of Sines, or  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$  can be used to determine the unknown side length or the unknown angle measures in any triangle.

### Example



In  $\triangle ABC$ , the measure of  $\angle A$  is  $65^\circ$ , the measure of angle  $B$  is  $80^\circ$ , and the length of side  $a$  is 7 feet. Use the Law of Sines to calculate the length of side  $b$ .

$$\frac{7}{\sin 65^\circ} = \frac{b}{\sin 80^\circ}$$

The length of side  $b$  is 7.6063 feet.

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## logarithm

The logarithm of a positive number is the exponent to which the base must be raised to result in that number.

### Example

Because  $10^2 = 100$ , the logarithm of 100 to the base 10 is 2.

$$\log 100 = 2$$

Because  $2^3 = 8$ , the logarithm of 8 to the base 2 is 3.

$$\log_2 (8) = 3$$

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## logarithm with same base and argument

The logarithm of a number, with the base equal to the same number, is always equal to 1.

$$\log_b (b) = 1$$

### Example

$$\log_4 (4) = 1$$

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## logarithmic equation

A logarithmic equation is an equation that contains a logarithm.

### Example

The equation  $\log_2(x) = 4$  is a logarithmic equation.

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## logarithmic function

A logarithmic function is a function involving a logarithm.

### Example

The function  $f(x) = 3 \log x$  is a logarithmic function.

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## M

## margin of error

The margin of error expresses the maximum expected difference between the true population data and the sample estimate of the data.

### Example

In a poll of 1100 registered voters, 54% said they would vote to re-elect the current mayor. The margin of error for the poll is  $\pm 3\%$ , which means that somewhere between 51% ( $54\% - 3\%$ ) and 57% ( $54\% + 3\%$ ) will actually vote to re-elect the current mayor.

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## mean ( $\mu$ )

The mean of a data set is the sum of all of the values of the data set divided by the number of values in the data set. The mean is also called the average.

### Example

The mean of the numbers 7, 9, 13, 4, and 7 is  $\frac{7 + 9 + 13 + 4 + 7}{5}$ , or 8.

The mean of a set of normally distributed data is aligned with the peak of the normal curve.

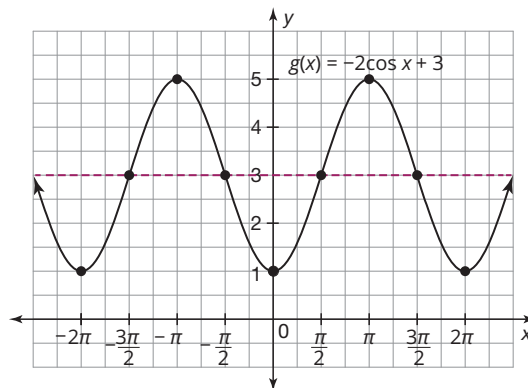
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## midline

The midline of a periodic function is a reference line whose equation is the average of the minimum and maximum values of the function.

### Example

In the graph of  $g(x) = -2 \cos x + 3$  the midline occurs at  $y = 3$  because the maximum value is 5 and the minimum value is 1.



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## multiplicity

Multiplicity is how many times a particular number is a zero for a given function.

### Example

The equation  $x^2 + 2x + 1 = 0$  has a double root at  $x = -1$ . The root  $-1$  has a multiplicity of 2.

$$x^2 + 2x + 1 = 0$$

$$(x + 1)(x + 1) = 0$$

$$x + 1 = 0 \quad \text{or} \quad x + 1 = 0$$

$$x = -1 \quad \text{or} \quad x = -1$$

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## N

## natural base $e$

The natural base  $e$  is an irrational number equal to approximately 2.71828.

### Example

$$e^2 \approx 2.7183^2 \approx 7.3892$$

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## natural logarithm

A natural logarithm is a logarithm with a base of  $e$ . Natural logarithms are usually written as  $\ln$ .

### Example

$\log_e(x)$  or  $\ln x$  is a natural logarithm.

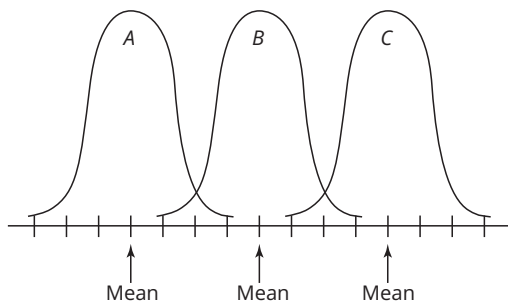
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## normal curve

A normal curve is a curve that is bell-shaped and symmetric about the mean.

### Example

A normal curve is shown.



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## normal distribution

A normal distribution, or normal probability distribution, describes a continuous data set that can be modeled using a normal curve.

### Example

Adult IQ scores, gas mileage of certain cars, and SAT scores are all continuous data that follow a normal distribution.

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## observational study

An observational study gathers data about a characteristic of the population without trying to influence the data.

### Example

The following is an example of an observational study. New research funded by a pediatric agency found that nearly 70% of in-house day care centers show as much as 2.5 hours of television to the children in the center per day. The study examined 132 day care programs in 2 Midwestern states.

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## odd function

An odd function  $f$  is a function for which  $f(-x) = -f(x)$  for all values of  $x$  in the domain.

### Example

The function  $f(x) = x^3$  is an odd function because  $(-x)^3 = -x^3$ .

---

---

## parameter

When data are gathered from a population, the characteristic used to describe the population is called a parameter.

### Example

If you wanted to find out the average height of the students at your school, and you measured every student at the school, the characteristic "average height" would be a parameter.

---

---

## percentile

A percentile is a data value for which a certain percent of the data is below the data value in a normal distribution.

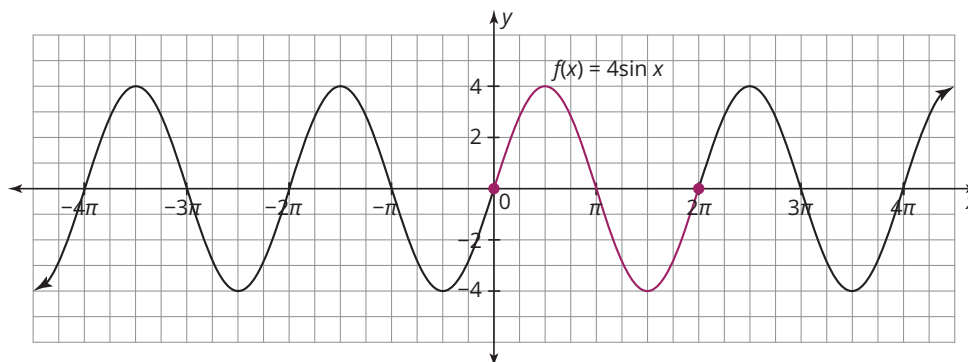
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## period

A period of a periodic function is the length of the smallest interval over which the function repeats.

### Example



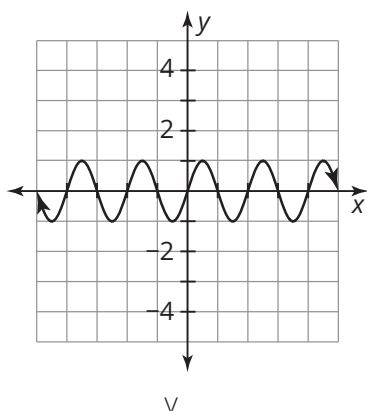
---

## periodic function

A periodic function is a function whose graph consists of repeated instances of a portion of the graph.

### Example

The function  $f(x) = \sin x$  is a periodic function. The portion of the graph between  $x = 0$  and  $x = 2\pi$  repeats.



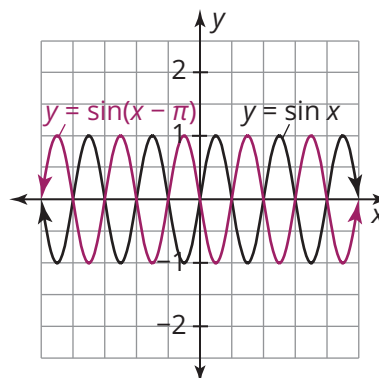
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## phase shift

A phase shift of a periodic function is a horizontal translation.

### Example

The function  $y = \sin(x - \pi)$  has a phase shift of  $\pi$  units from the basic function  $y = \sin x$ .



---

## periodicity identity

A periodicity identity is a trigonometric identity based on the period of the trigonometric functions.

### Example

The six periodicity identities are:

$$\sin(x + 2\pi) = \sin x; \cos(x + 2\pi) = \cos x$$

$$\sec(x + 2\pi) = \sec x; \csc(x + 2\pi) = \csc x$$

$$\tan(x + \pi) = \tan x; \cot(x + \pi) = \cot x$$

---

## polynomial function

A polynomial function is a function that can be written in the form

$p(x) = \blacksquare x^n + \blacksquare x^{n-1} + \cdots + \blacksquare x^2 + \blacksquare x + \blacksquare$ , where the coefficients, represented by each  $\blacksquare$ , are complex numbers and the exponents are nonnegative integers.

### Example

The function  $f(x) = 5x^3 + 3x^2 + x + 1$  is a polynomial function.

---

## polynomial long division

Polynomial long division is an algorithm for dividing one polynomial by another of equal or lesser degree.

### Example

$$\begin{array}{r} 4x^2 - 6x + 3 \\ 2x + 3 \overline{) 8x^3 + 0x^2 - 12x - 7} \\ \underline{-(8x^3 + 12x^2)} \phantom{- 7} \\ -12x^2 - 12x \phantom{- 7} \\ \underline{-(-12x^2 - 18x)} \phantom{- 7} \\ 6x - 7 \\ \underline{-(6x + 9)} \\ \text{Remainder } \textcircled{-16} \end{array}$$

---

## population

The population is the entire set of items from which data can be selected. When you decide what you want to study, the population is the set of all elements in which you are interested. The elements of that population can be people or objects.

### Example

If you wanted to find out the average height of the students at your school, the number of students at the school would be the population.

---

## population proportion

A population proportion is the percentage of an entire population that yields a favorable outcome in an experiment.

### Example

In an election, the population proportion represents the percentage of people in the entire town who vote to re-elect the mayor.

---

## power function

A power function is a function of the form  $P(x) = ax^n$  where  $n$  is a non-negative integer.

### Example

The functions  $f(x) = x$ ,  $f(x) = x^2$ , and  $f(x) = x^3$  are power functions.

---

## Power Rule of Logarithms

The Power Rule of Logarithms states that the logarithm of a power is equal to the product of the exponent and the logarithm of the base of the power.

$$\log_b(x)^n = n \cdot \log_b(x)$$

### Example

$$\ln(x)^2 = 2 \ln x$$

---

## Product Rule of Logarithms

The Product Rule of Logarithms states that the logarithm of a product is equal to the sum of the logarithms of the factors.

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

### Example

$$\log(5x) = \log 5 + \log x$$

---

## Pythagorean identity

A Pythagorean identity is a trigonometric identity based on the Pythagorean Theorem.

### Example

The three Pythagorean identities are:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

---

## Q

---

### Quadratic Formula

The Quadratic Formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , can be used to calculate the solutions to any quadratic equation of the form  $ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  represent real numbers and  $a \neq 0$ .

---

### quartic function

A quartic function is a polynomial function with a degree of four.

#### Examples

The function  $f(x) = 3x^4 - 2x + 5$  is a quartic function.

---

### quintic function

A quintic function is a polynomial function with a degree of five.

#### Examples

The function  $f(x) = 5x^5 + 3x^4 + x^3$  is a quintic function.

---

### Quotient Rule of Logarithms

The Quotient Rule of Logarithms states that the logarithm of a quotient is equal to the difference of the logarithms of the dividend and the divisor.

$$\log_b \left( \frac{x}{y} \right) = \log_b (x) - \log_b (y)$$

#### Examples

$$\log \left( \frac{x}{2} \right) = \log x - \log 2$$

---

## R

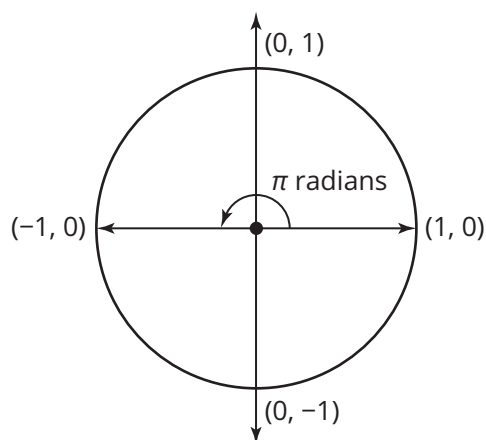
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### radians

A radian is a unit of measurement for an angle in standard position. The ratio of the intercepted arc length of a central angle to the length of the radius is the measure of the central angle in radians.

#### Example

The angle shown has a radian measure of  $\pi$  radians.



---

### radical function

A radical function is a function that contains one or more radical expressions.

#### Example

The function  $f(x) = \sqrt{3x + 5}$  is a radical function.

---

### random sample

A random sample is a method of collecting data in which every member of a population has an equal chance of being selected.

#### Example

Choosing 100 fans at random to participate in a survey from a crowd of 5000 people is an example of a random sample.



---

## rational equation

A rational equation is an equation that contains one or more rational expressions.

### Example

The equation  $\frac{1}{x-1} + \frac{1}{x+1} = 4$  is a rational equation.

---

## rational function

A rational function is any function that can be written as the ratio of two polynomial functions. A rational function can be written in the form  $f(x) = \frac{P(x)}{Q(x)}$  where  $P(x)$  and  $Q(x)$  are polynomial functions, and  $Q(x) \neq 0$ .

### Example

The function  $f(x) = \frac{1}{x-1} + \frac{1}{x+1}$  is a rational function.

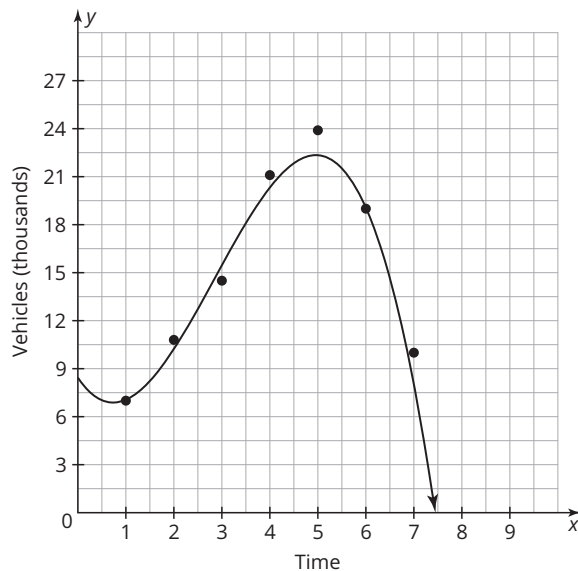
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## regression equation

A regression equation is a function that models the relationship between two variables in a scatter plot.

### Example

The regression equation  $y = -0.41x^3 + 3.50x^2 - 4.47x + 8.44$  models the relationship between time and the number of vehicles.



---

## relation

A relation is the mapping between a set of input values called the domain and a set of output values called the range.

### Example

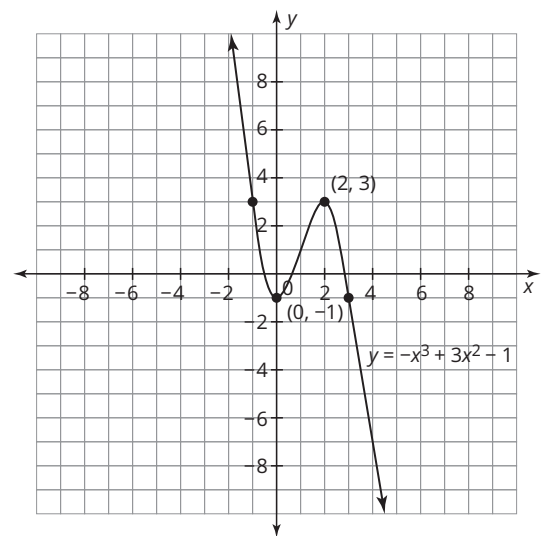
The set of points  $\{(0, 1), (1, 8), (2, 5), (3, 7)\}$  is a relation.

---

## relative maximum

A relative maximum is the highest point in a particular section of a graph.

### Example



The graph shown has a relative maximum at (2, 3).

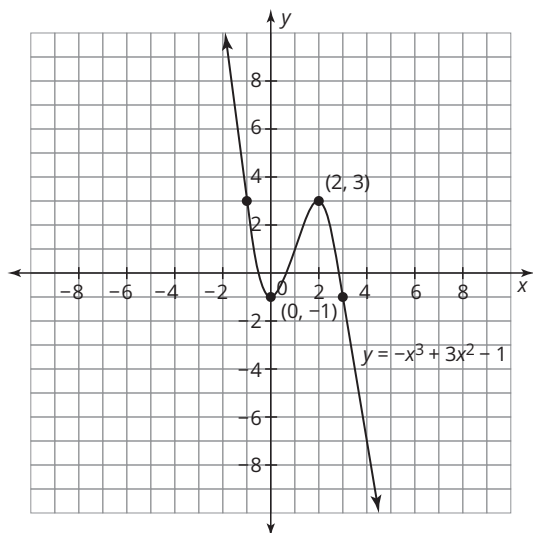
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## relative minimum

A relative minimum is the lowest point in a particular section of a graph.

### Example

The graph shown has a relative minimum at  $(0, -1)$ .



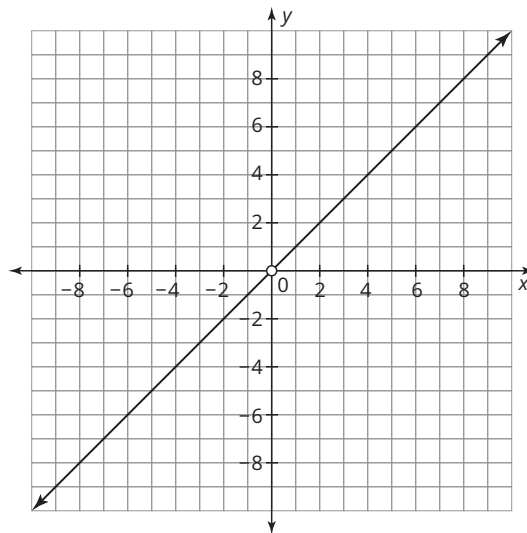
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## removable discontinuity

A removable discontinuity is a single point at which the graph of a function is not defined.

### Example

The graph of the function  $f(x) = \frac{x^2}{x}$  has a removable discontinuity at  $x = 0$ .



---

## Remainder Theorem

The Remainder Theorem states that the remainder when dividing a polynomial by  $(x - r)$  is the value of the polynomial at  $r$ .

### Example

The value of the polynomial  $x^2 + 5x + 2$  at 1 is  $(1)^2 + 5(1) + 2 = 8$ . So, the remainder when  $x^2 + 5x + 2$  is divided by  $x - 1$  is 8.

$$\begin{array}{r} x + 6 \\ x - 1 \overline{) x^2 + 5x + 2} \\ \underline{x^2 - x} \phantom{+ 2} \\ 6x + 2 \\ \underline{6x - 6} \\ 8 \end{array}$$

---

## S

### sample

Where data are collected from a selection of the population, the data are called a sample.

### Example

If you wanted to find out the average height of the students in your school, you could choose just a certain number of students and measure their heights. The heights of the students in this group would be the sample.

## sample proportion

A sample proportion is the percentage of a sample that yields a favorable outcome in an experiment. This is often used to make predictions about a population.

### Example

In an election, a sample of townspeople is surveyed. The sample proportion represents the percentage of the survey results that indicate that they will vote to re-elect the mayor.

## sample survey

A sample survey poses a question of interest to a random sample of the targeted population.

### Example

The following is an example of a sample survey. A recent survey of nearly 1200 young people from across the U.S. shows that 40% of 16- to 20-year-olds who have a driver's license admit to texting on a regular basis while they are driving.

## sampling distribution

A sampling distribution consists of every possible sample of equal size from a given population. A sampling distribution provides an estimate for population parameters. The mean or proportion of a sampling distribution is estimated by the mean or proportion of a sample. For categorical data, the standard deviation of a sampling distribution is estimated

by calculating  $\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$  where  $\hat{p}$  (p-hat) is the sample proportion and  $n$  is the sample size.

For continuous data, the standard deviation of a sampling distribution is estimated by

calculating  $\frac{s}{\sqrt{n}}$  where  $s$  is the standard deviation of the original sample and  $n$  is the sample size.

### Example

A sleep survey of 50 teens resulted in a sample mean of 7.7 hours and sample standard deviation of 0.8 hours.

The estimated mean of the sampling distribution is 7.7 hours. The estimated standard deviation of the sampling distribution is approximately 0.11 hour.

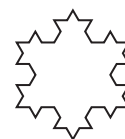
$$\frac{s}{\sqrt{n}} = \frac{0.8}{\sqrt{50}} \approx 0.11$$

## self-similar

A self-similar object is exactly or approximately similar to a part of itself.

### Example

A Koch snowflake is considered to be self-similar.



## simple random sample

A simple random sample is a sample in which every member of the population has the same chance of being selected.

### Example

Using a random number generator to select a sample is an example of simple random sampling.

---

## sine function

The sine function is a periodic function. It takes angle measures ( $\theta$  values) as inputs and then outputs real number values which correspond to the coordinates of points on the unit circle.

### Example

The function  $h(\theta) = -\sin(2\theta) + 1$  is a sine function.

---

## square root function

The square root function is the inverse of the power function  $f(x) = x^2$  when the domain is restricted to  $x \geq 0$ .

### Example

The square root function is  $g(x) = \sqrt{x}$ .

---

## standard deviation ( $\sigma$ )

Standard deviation is a measure of the variation of the values in a data set from the mean of the data. A lower standard deviation represents data that are more tightly clustered near the mean. A higher standard deviation represents data that are more spread out from the mean. Use the formula below to calculate standard deviation.

$$\text{standard deviation} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$$

where  $\bar{x}$  is the mean and  $n$  is the number of data values in the data set  $\{x_1, x_2, \dots, x_n\}$ .

### Example

In the data set of test scores 60, 70, 80, 90, 100, the mean  $\bar{x}$  is 80 and the number of data elements  $n$  is 5.

$$\begin{aligned}\sigma &= \sqrt{\frac{(60 - 80)^2 + (70 - 80)^2 + (80 - 80)^2 + (90 - 80)^2 + (100 - 80)^2}{5}} \\ &= \sqrt{\frac{1000}{5}} \\ &= \sqrt{200} \\ &\approx 14.14.\end{aligned}$$

---

## standard form (general form) of a quadratic function

A quadratic function written in the form  $f(x) = ax^2 + bx + c$ , where  $a \neq 0$ , is in standard form, or general form.

### Example

The function  $f(x) = -5x^2 - 10x + 1$  is written in standard form.

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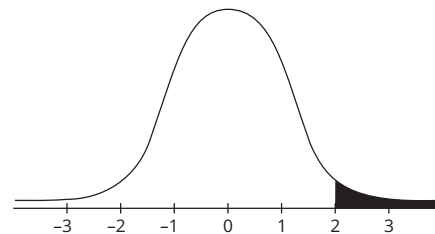
## standard normal distribution

The standard normal distribution is a normal probability distribution with the following properties:

- The mean is equal to 0.
- The standard deviation is 1.
- The curve is bell-shaped and symmetric about the mean.

### Example

A standard normal distribution curve is shown.



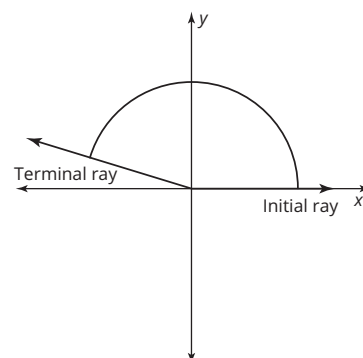
---

## standard position of an angle

The standard position of an angle occurs when the vertex of the angle is at the origin and one ray of the angle is on the x-axis.

### Example

The angle shown is in standard position.



---

## statistic

When data are gathered from a sample, the characteristic used to describe the sample is called a statistic.

### Example

If you wanted to find out the average height of the students in your school, and you chose just a certain number of students randomly and measured their heights, the characteristic “average height” would be called a statistic.

---

## statistically significant

A survey that has a result that is statistically significant indicates that the result did not likely occur by chance, but is likely linked to a specific cause. Typically, a result that is more than 2 standard deviations away from the mean is considered statistically significant.

### Example

A survey of 2000 teenagers reports that 42% have a part-time job. The interval from 40.9% to 43.1% represents a 95% confidence interval for the population proportion. A survey that yields a report of 50% of teenagers with a part-time job would be considered statistically significant.

---

## stratified random sample

A stratified random sample is a sample obtained by dividing the population into different groups, or strata, according to a characteristic, and randomly selecting data from each group.

### Example

If students in a high school are divided by class, and random samples are then taken from each class, the result is a stratified random sample.

---

## subjective sample

A subjective sample is a sample that is chosen based on some criteria, rather than at random.

### Example

From a set of students, “choosing five students you know” is a subjective sample. In contrast, “choosing five students at random” is a random sample.

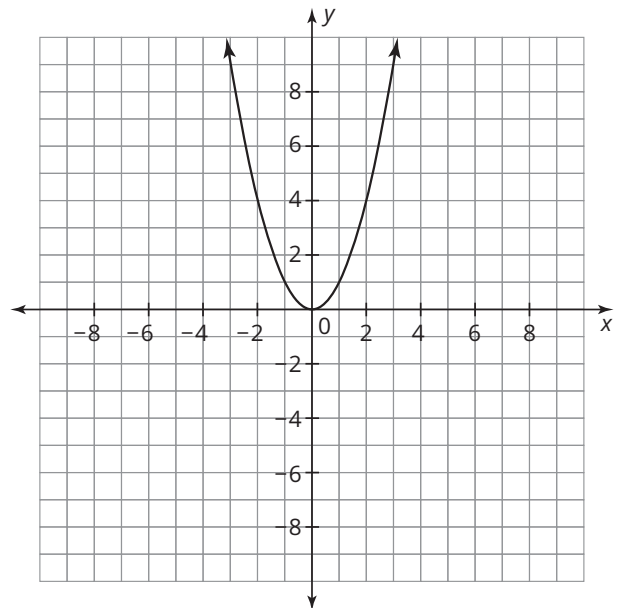
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## symmetric about a line

If a graph is symmetric about a line, the line divides the graph into two identical parts.

### Example

The graph of  $f(x) = x^2$  is symmetric about the line  $x = 0$ .



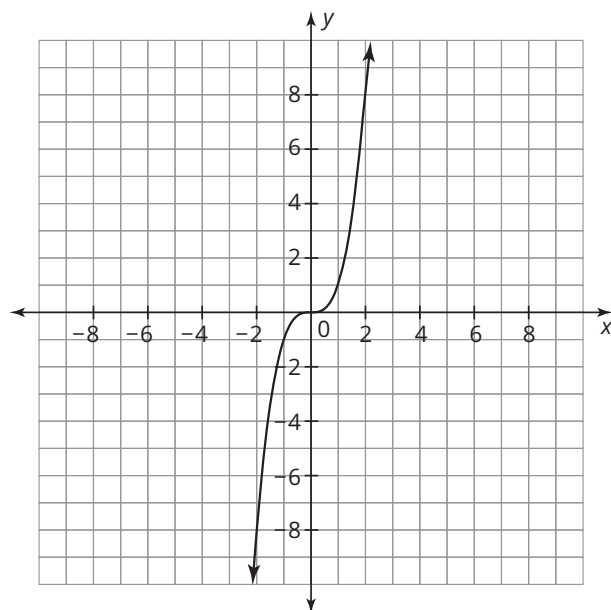
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## symmetric about a point

A function is symmetric about a point if each point on the graph has a point the same distance from the central point, but in the opposite direction.

### Example

The graph of  $f(x) = x^3$  is symmetric about the point  $(0, 0)$ .



---

## synthetic division

Synthetic division is a method for dividing a polynomial by a linear factor of the form  $(x - r)$ .

### Example

The quotient of  $2x^2 - 3x - 9$  and  $x - 3$  can be calculated using synthetic division.

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -9 & \\ & & 6 & 9 & \\ \hline & 2 & 3 & 0 & \end{array}$$

The quotient of  $2x^2 - 3x - 9$  and  $x - 3$  is  $2x + 3$ .

---

## systematic sample

A systematic sample is a sample obtained by selecting every  $n$ th data in the population.

### Example

If you choose every 12th student that walks into school, your sample is a systematic sample.

---

## T

## tangent function

The tangent function is a periodic function. It takes angle measures ( $\theta$  values) as inputs and then outputs real number values which correspond to the coordinates of points on the unit circle.

### Example

The function  $f(\theta) = \tan\left(\frac{\theta}{2}\right)$  is a tangent function.

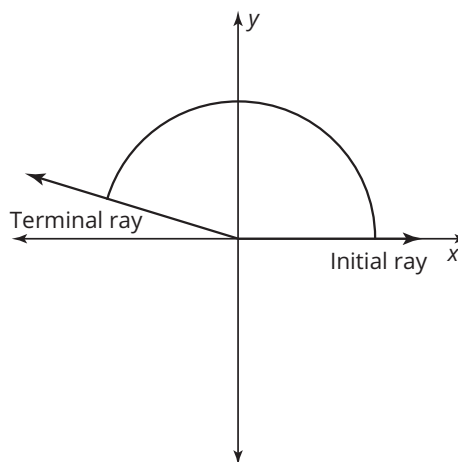
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## terminal ray of an angle

The terminal ray of an angle in standard position is the ray with its endpoint at the origin that is not the initial ray.

### Example

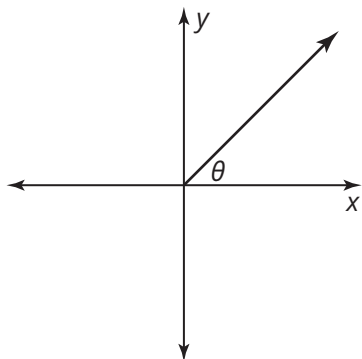
The terminal ray of the angle is labeled in the diagram.



theta ( $\theta$ )

Theta is a symbol typically used to represent the measure of an angle in standard position.

Example



treatment

A treatment is a condition in an experiment.

Example

Suppose that an experiment is conducted to test the effects of a new drug on a sample of patients. The distribution of the drug to the patients is the treatment in the experiment.

trigonometric equation

A trigonometric equation is an equation that includes one or more trigonometric functions.

Example

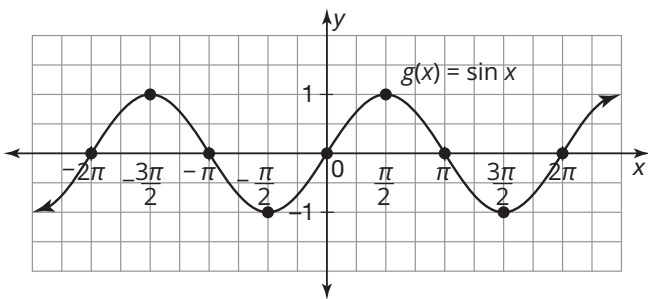
The equation  $\cos x = \frac{\sqrt{2}}{2}$  is a trigonometric equation.

trigonometric function

A trigonometric function is a periodic function that takes angle measures ( $\theta$  values) as inputs and then outputs real number values which correspond to the coordinates of points on the unit circle.

Example

The function  $g(x) = \sin x$  is a trigonometric function. The graph of the sine function  $g(\theta) = \sin \theta$  is obtained by evaluating the  $\theta$  values of the unit circle and graphing the coordinates.



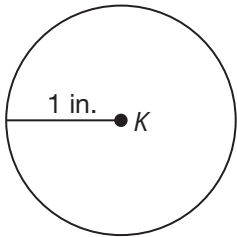
$\theta$	$g(\theta) = \sin \theta$	$(\theta, g(\theta))$
0	$\sin(0) = 0$	$(0, 0)$
$\frac{\pi}{2}$	$\sin\left(\frac{\pi}{2}\right) = 1$	$\left(\frac{\pi}{2}, 1\right)$
$\pi$	$\sin(\pi) = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$\sin\left(\frac{3\pi}{2}\right) = -1$	$\left(\frac{3\pi}{2}, -1\right)$
$2\pi$	$\sin(2\pi) = 0$	$(2\pi, 0)$

U

unit circle

A unit circle is a circle whose radius is one unit of distance.

Example



Circle  $K$  is a unit circle.

---

## V

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### vertex form of a quadratic function

A quadratic function written in vertex form is in the form  $f(x) = a(x - h)^2 + k$ , where  $a \neq 0$ .

#### Example

The quadratic equation  $y = 2(x - 5)^2 + 10$  is written in vertex form. The vertex of the graph is the point (5, 10).

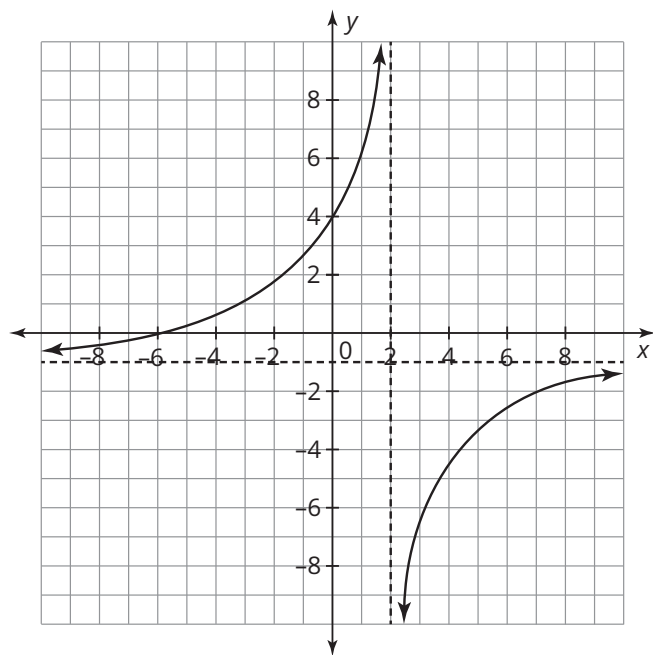
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### vertical asymptote

A vertical asymptote is a vertical line that a function gets closer and closer to, but never intersects. The asymptote does not represent points on the graph of the function. It represents the output value that the graph approaches.

#### Example

The graph has two asymptotes: a vertical asymptote  $x = 2$  and a horizontal asymptote  $y = -1$ .



---

## volunteer sample

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A volunteer sample is a sample whose data consists of those who volunteer to be part of the sample.

#### Example

If you ask students in your school to complete and submit an optional survey so that you can collect data, your sample is a volunteer sample.

---

## Z

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### z-score

A z-score is a number that describes how many standard deviations from the mean a particular value is. The following formula can be used to calculate a z-score for a particular value, where  $z$  represents the z-score,  $x$  represents the particular data value,  $\mu$  represents the mean, and  $\sigma$  represents the standard deviation.

$$z = \frac{x - \mu}{\sigma}$$

#### Example

Suppose that a set of data follows a normal distribution with a mean of 22 and a standard deviation of 2.4.

The z-score for a data value of 25 is  $z = \frac{25 - 22}{2.4} = 1.25$ .

---

### Zero Product Property

The Zero Product Property states: "If the product of two or more factors is equal to zero, then at least one factor must be equal to zero." This is also called the Converse of Multiplication Property of Zero.

#### Example

According to the Zero Product Property, if  $(x - 2)(x + 3) = 0$  then  $x - 2 = 0$  or  $x + 3 = 0$ .

---

### Zero Property of Logarithms

The Zero Property of Logarithms states that the logarithm of 1, with any base, is always equal to 0.

$$\log_b(1) = 0$$

#### Example

$$\log_3(1) = 0$$