## Glossary

## A

## Addition Property of Equality

The addition property of equality states:
"If $a=b$, then $a+c=b+c$."

## Example

If $x=2$, then $x+5=2+5$, or $x+5=7$ is an example of the Addition Property of Equality.

## Addition Rule for Probability

The Addition Rule for Probability states: "The probability that Event $A$ occurs or Event $B$ occurs is the probability that Event $A$ occurs plus the probability that Event $B$ occurs minus the probability that both $A$ and $B$ occur."

$$
P(A \text { or } B)=P(A)+P(B)=P(A \text { and } B)
$$

## Example

You flip a coin two times. Calculate the probability of flipping a heads on the first flip or flipping a heads on the second flip.
Let $A$ represent the event of flipping a heads on the first flip. Let $B$ represent the event of flipping a heads on the second flip.
$P(A$ or $B)=P(A)+P(B)-P(A$ and $B)$
$P(A$ or $B)=\frac{1}{2}+\frac{1}{2}-\frac{1}{4}$
$P(A$ or $B)=\frac{3}{4}$
So, the probability of flipping a heads on the first flip or flipping a heads on the second flip is $\frac{3}{4}$.

## adjacent angles

Adjacent angles are angles that share a common side and a common vertex, and lie on opposite sides of their common side.

## Example

Angle $B A C$ and angle $C A D$ are adjacent angles. Angle FEG and angle GEH are adjacent angles.



## adjacent arcs

Adjacent arcs are two arcs of the same circle sharing a common endpoint.

## Example

Arcs $Z A$ and $A B$ are adjacent arcs.


## adjacent side

The adjacent side of a triangle is the side adjacent to the reference angle that is not the hypotenuse.

## Example



## altitude

An altitude is a line segment drawn from a vertex of a triangle perpendicular to the line containing the opposite side.

## Example

Segment $E G$ is an altitude of triangle $F E D$.


## angle

An angle is a figure that is formed by two rays that extend from a common point called the vertex.

## Example

Angles $A$ and $B$ are shown.


## angle bisector

An angle bisector is a ray that divides an angle into two angles of equal measure.

## Example

Ray $A T$ is the angle bisector of angle $M A H$.


## angular velocity

Angular velocity is a type of circular velocity described as an amount of angle movement in radians over a specified amount of time. Angular velocity can be expressed as $\omega=\frac{\theta}{t}$, where $\omega=$ angular velocity, $\theta=$ angular measurement in radians, and $t=$ time .

## annulus

An annulus is the region bounded by two concentric circles.

## Example

The annulus is the shaded region shown.


## arc

An arc is the curve between two points on a circle. An arc is named using its two endpoints.

## Example

The symbol used to describe $\operatorname{arc} B C$ is $\overparen{B C}$.


## arc length

An arc length is a portion of the circumference of a circle. The length of an arc of a circle can be calculated by multiplying the circumference of the circle by the ratio of the measure of the arc to $360^{\circ}$.

$$
\text { arc length }=2 \pi r \cdot \frac{x^{\circ}}{360^{\circ}}
$$

## Example

In circle $A$, the radius $\overline{A B}$ is 3 centimeters and the measure of arc $B C$ is 83 degrees.
$(2 \pi r)\left(\frac{m \overparen{B C}}{360^{\circ}}\right)=2 \pi(3)\left(\frac{83^{\circ}}{360^{\circ}}\right)$

$$
\approx 4.35
$$

So, the length of $\operatorname{arc} B C$ is approximately 4.35 centimeters.


## axis of symmetry

An axis of symmetry is a line that passes through a figure and divides the figure into two symmetrical parts that are mirror images of each other.

## Example

Line $k$ is the axis of symmetry of the parabola.


## B

## base angles of a trapezoid

The base angles of a trapezoid are either pair of angles that share a base as a common side.

## Example

Angle $T$ and angle $R$ are one pair of base angles of trapezoid $P A R T$. Angle $P$ and angle $A$ are another pair of base angles.


## biconditional statement

A biconditional statement is a statement written in the form "if and only if $p$, then $q$." It is a combination of both a conditional statement and the converse of that conditional statement. A biconditional statement is true only when the conditional statement and the converse of the statement are both true.

## Example

Consider the property of an isosceles trapezoid: "The diagonals of an isosceles trapezoid are congruent." The property states that if a trapezoid is isosceles, then the diagonals are congruent. The converse of this statement is true: "If the diagonals of a trapezoid are congruent, then the trapezoid is an isosceles trapezoid." So, this property can be written as a biconditional statement: "A trapezoid is isosceles if and only if its diagonals are congruent."

## binomial

Polynomials with exactly two terms are binomials.

## Example

The polynomial $3 x+5$ is a binomial.


## categorical data (qualitative data)

Categorical data are data that each fit into exactly one of several different groups, or categories. Categorical data are also called "qualitative data."

## Example

Animals: lions, tigers, bears, etc.
U.S. Cities: Los Angeles, Atlanta, New York City, Dodge City, etc.

The set of animals and the set of U.S. cities are two examples of categorical data sets.

Cavalieri's principle
Cavalieri's principle states that if all one-dimensional
slices of two-dimensional figures have the same lengths,
then the two-dimensional figures have the same area.
The principle also states that given two solid figures
included between parallel planes, if every plane cross
section parallel to the given planes has the same area in
both solids, then the volumes of the solids are equal.
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both solids, then the volumes of the solids are equal.

## center of a circle

The center of a circle is a fixed point in the plane that is at an equal distance from every point on the circle.

## Example

Point $H$ is the center of the circle.


## central angle

A central angle of a circle is an angle whose sides are radii. The measure of a central angle is equal to the measure of its intercepted arc.

## Example

In circle $O, \angle A O C$ is a central angle and $\overparen{A C}$ is its intercepted arc. If $m \angle A O C=45^{\circ}$, then $m \overparen{A C}=45^{\circ}$.


## centroid

The centroid of a triangle is the point at which the medians of the triangle intersect.

## Example

Point $X$ is the centroid of triangle $A B C$.


## chord

A chord is a line segment whose endpoints are points on a circle. A chord is formed by the intersection of the circle and a secant line.

## Example

Segment $C D$ is a chord of circle $O$.


## circular permutation

A circular permutation is a permutation in which there is no starting point and no ending point. The circular permutation of $n$ objects is $(n-1)$ !.

## Example

A club consists of four officers: a president (P), a vicepresident (VP), a secretary (S), and a treasurer (T). There are (4-1)!, or 6 ways for the officers to sit around a round table.

## circumcenter

The circumcenter of a triangle is the point at which the perpendicular bisectors intersect.

## Example

Point $X$ is the circumcenter of triangle $A B C$.


## circumscribed polygon

A circumscribed polygon is a polygon drawn outside a circle such that each side of the polygon is tangent to the circle.

## Example

Triangle $A B C$ is a circumscribed triangle.


## closed (closure)

When an operation is performed on any of the numbers in a set and the result is a number that is also in the same set, the set is said to be closed (or to have closure) under that operation.

## Example

The set of whole numbers is closed under addition. The sum of any two whole numbers is always another whole number.

## closed interval

A closed interval $[a, b]$ describes the set of all numbers between $a$ and $b$, including $a$ and $b$.

## Example

The interval $[3,7]$ is the set of all numbers greater than or equal to 3 and less than or equal to 7 .

## coefficient

Within a polynomial, a coefficient is a number multiplied by a power.

## Example

The term $3 x^{5}$ has a coefficient of 3 .

## coefficient of determination

The coefficient of determination measures the "strength" of the relationship between the original data and its quadratic regression equation.

## collinear points

Collinear points are points that are located on the same line.

## Example

Points $A, B$, and $C$ are collinear.


## combination

A combination is an unordered collection of items. One notation for the combinations of $r$ elements taken from a collection of $n$ elements is:

$$
{ }_{n} C_{r}=C(n, r)=C_{r}^{n}
$$

## Example

The two-letter combinations of the letters $A, B$, and $C$ are: $A B, A C, B C$.

## compass

A compass is a tool used to create arcs and circles.

## Example



## complement of an event

The complement of an event is an event that contains all the outcomes in the sample space that are not outcomes in the event. In mathematical notation, if $E$ is an event, then the complement of $E$ is often denoted as $\bar{E}$ or $E^{c}$.

## Example

A number cube contains the numbers 1 though 6 . Let $E$ represent the event of rolling an even number. The complement of Event $E$ is rolling an odd number.

## complementary angles

Two angles are complementary if the sum of their measures is $90^{\circ}$.

## Example

Angle 1 and angle 2 are complementary angles. $m \angle 1+m \angle 2=90^{\circ}$


## completing the square

Completing the square is a process for writing a quadratic expression in vertex form which then allows you to solve for the zeros.

## complex conjugates

Complex conjugates are pairs of numbers of the form $a+b i$ and $a-b i$. The product of a pair of complex conjugates is always a real number.

## Example

The expressions $(1+i)$ and $(1-i)$ are complex conjugates. The product of $(1+i)$ and $(1-i)$ is a real number: $(1+i)(1-i)=1-i^{2}=1-(-1)=2$.

## complex numbers

The set of complex numbers is the set of all numbers written in the form $a+b i$, where $a$ and $b$ are real numbers.

## composition of functions

A composition of functions is the combination of functions such that the output from one function becomes the input for the next function.

## Example

The composition of function $f(x)$ composed with $g(x)$ is denoted $(f \circ g)(x)$ or $f(g(x))$. It is read as " $f$ composed with $g(x)$ " or " $f$ of $g(x)$."

## compound event

A compound event combines two or more events, using the word "and" or the word "or."

## Example

You roll a number cube twice. Rolling a six on the first roll and rolling an odd number on the second roll are compound events.

## concavity

The concavity of a parabola describes the orientation of the curvature of the parabola.

## Example



## concentric circles

Concentric circles are circles in the same plane that have a common center.

## Example

The circles shown are concentric because they are in the same plane and have a common center $H$.


## conclusion

Conditional statements are made up of two parts. The conclusion is the result that follows from the given information.

## Example

In the conditional statement "If two positive numbers are added, then the sum is positive," the conclusion is "the sum is positive."

## concurrent

Concurrent lines, rays, or line segments are three or more lines, rays, or line segments intersecting at a single point.

## Example

Lines $\ell, m$, and $n$ are concurrent lines.


## conditional probability

A conditional probability is the probability of event $B$, given that event $A$ has already occurred. The notation for conditional probability is $P(B \mid A)$, which reads, "the probability of event $B$, given event $A$."

## Example

The probability of rolling a 4 or less on the second roll of a number cube, given that a 5 is rolled first, is an example of a conditional probability.

## conditional statement

A conditional statement is a statement that can be written in the form "If $p$, then $q$."

## Example

The statement "If I close my eyes, then I will fall asleep" is a conditional statement.

## congruent line segments

Congruent line segments are two or more line segments that have equal measures.

## Example

Line segment $A B$ is congruent to line segment $C D$.


## conjecture

A conjecture is a hypothesis that something is true. The hypothesis can later be proved or disproved.

## construct

A constructed geometric figure is created using only a compass and a straightedge.

## construction proof

A construction proof is a proof that results from creating a figure with specific properties using only a compass and straightedge.

## Example

A construction proof is shown of the conditional statement: If $\overline{A B} \cong \overline{C D}$, then $\overline{A C} \cong \overline{B D}$.


## contrapositive

To state the contrapositive of a conditional statement, negate both the hypothesis and the conclusion and then interchange them.

Conditional Statement: If $p$, then $q$.
Contrapositive: If not $q$, then not $p$.

## Example

Conditional Statement: If a triangle is equilateral, then it is isosceles.
Contrapositive: If a triangle is not isosceles, then it is not equilateral.

## converse

To state the converse of a conditional statement, interchange the hypothesis and the conclusion.

Conditional Statement: If $p$, then $q$.
Converse: If $q$, then $p$.

## Example

Conditional Statement: If $a=0$ or $b=0$, then $a b=0$.
Converse: If $a b=0$, then $a=0$ or $b=0$.

## Converse of Multiplication Property of Zero

The Converse of Multiplication Property of Zero states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero. This is also called the Zero Product Property.

## Example

If $(x-2)(x+3)=0$, then $x-2=0$ or $x+3=0$.

## coplanar lines

Coplanar lines are lines that lie in the same plane.

## Example

Line $A$ and line $B$ are coplanar lines. Line $C$ and line $D$ are not coplanar lines.


## corresponding parts of congruent triangles are congruent (CPCTC)

CPCTC states that if two triangles are congruent, then each part of one triangle is congruent to the corresponding part of the other triangle.

## Example

In the triangles shown, $\triangle X Y Z \cong \triangle L M N$. Because corresponding parts of congruent triangles are congruent (CPCTC), the following corresponding parts are congruent.

- $\angle X \cong \angle L$
- $\angle Y \cong \angle M$
- $\angle Z \cong \angle N$
- $\overline{X Y} \cong \overline{L M}$
- $\overline{Y Z} \cong \overline{M N}$
- $\overline{X Z} \cong \overline{L N}$



## cosecant (csc)

The cosecant (csc) of an acute angle in a right triangle is the ratio of the length of the hypotenuse to the length of the side opposite the angle.

## Example

In triangle $A B C$, the cosecant of angle $A$ is:
$\csc A=\frac{\text { length of hypotenuse }}{\text { length of side opposite } \angle A}=\frac{A B}{B C}$
The expression "csc $A$ " means "the cosecant of angle $A$."


## cosine (cos)

The cosine (cos) of an acute angle in a right triangle is the ratio of the length of the side adjacent to the angle to the length of the hypotenuse.

## Example

In triangle $A B C$, the cosine of angle $A$ is:
$\cos A=\frac{\text { length of side adjacent to } \angle A}{\text { length of hypotenuse }}=\frac{A C}{A B}$
The expression "cos $A$ " means "the cosine of angle $A$."


## cotangent (cot)

The cotangent (cot) of an acute angle in a right triangle is the ratio of the length of the side adjacent to the angle to the length of the side opposite the angle.

## Example

In triangle $A B C$, the cotangent of angle $A$ is:
$\cot A=\frac{\text { length of side adjacent to } \angle A}{\text { length of side opposite } \angle A}=\frac{A C}{B C}$
The expression "cot $A$ " means "the cotangent of angle $A$."


## counterexample

A counterexample is a single example that shows that a statement is not true.

## Example

Your friend claims that you add fractions by adding the numerators and then adding the denominators. A counterexample is $\frac{1}{2}+\frac{1}{2}$. The sum of these two fractions is 1. Your friend's method results in $\frac{1+1}{2+2}$, or $\frac{1}{2}$. Your friend's method is incorrect.

## Counting Principle

The Counting Principle states that if action $A$ can occur in $m$ ways and for each of these $m$ ways action $B$ can occur in $n$ ways, then actions $A$ and $B$ can occur in $m \cdot n$ ways.

## Example

In the school cafeteria, there are 3 different main entrées and 4 different sides. So, there are $3 \cdot 4$, or 12 different lunches that can be created.

## D

## deduction

Deduction is reasoning that involves using a general rule to make a conclusion.

## Example

Sandy learned the rule that the sum of the measures of the three interior angles of a triangle is 180 degrees. When presented with a triangle, she concludes that the sum of the measures of the three interior angles is 180 degrees. Sandy reached the conclusion using deduction.

## degree measure of an arc

The degree measure of a minor arc is equal to the degree measure of its central angle. The degree measure of a major arc is determined by subtracting the degree measure of the minor arc from $360^{\circ}$.

## Example

The measure of minor arc $A B$ is $30^{\circ}$. The measure of major arc $B Z A$ is $360^{\circ}-30^{\circ}=330^{\circ}$.


## degree of a polynomial

The greatest exponent in a polynomial determines the degree of the polynomial.

## Example

The polynomial $2 x^{3}+5 x^{2}-6 x+1$ has a degree of 3 .

## degree of a term

The degree of a term in a polynomial is the exponent of the term.

## Example

In the polynomial $5 x^{2}-6 x+9$, the degree of the term $6 x$ is 1 .

## dependent events

Dependent events are events for which the occurrence of one event has an impact on the occurrence of subsequent events.

## Example

A jar contains 1 blue marble, 1 green marble, and 2 yellow marbles. You randomly choose a yellow marble without replacing the marble in the jar, and then randomly choose a yellow marble again. The events of randomly choosing a yellow marble first and randomly choosing a yellow marble second are dependent events because the 1st yellow marble was not replaced in the jar.

## diameter

The diameter of a circle is a line segment with each endpoint on the circle that passes through the center of the circle.

## Example

In circle $O, \overline{A B}$ is a diameter.


## diameter of a sphere

The diameter of a sphere is a line segment with each endpoint on the sphere that passes through the center of the sphere.

## Example



## difference of two cubes

The difference of two cubes is an expression in the form $a^{3}-b^{3}$ that can be factored as $(a-b)\left(a^{2}+a b+b^{2}\right)$.

## Example

The expression $x^{3}-8$ is a difference of two cubes because it can be written in the form $x^{3}-2^{3}$. The expression can be factored as $(x-2)\left(x^{2}+2 x+4\right)$.

## difference of two squares

The difference of two squares is an expression in the form $a^{2}-b^{2}$ that can be factored as $(a+b)(a-b)$.

## Example

The expression $x^{2}-4$ is a difference of two squares because it can be written in the form $x^{2}-2^{2}$. The expression can be factored as $(x+2)(x-2)$.

## dilation factor

The dilation factor is the common factor which every $y$-coordinate of a graph is multiplied by to produce a vertical dilation.

## direct proof

A direct proof begins with the given information and works to the desired conclusion directly through the use of givens, definitions, properties, postulates, and theorems.

## directrix of a parabola

The directrix of a parabola is a line such that all points on the parabola are equidistant from the focus and the directrix.

## Example

The focus of the parabola shown is the point $(0,2)$. The directrix of the parabola shown is the line $y=-2$. All points on the parabola are equidistant from the focus and the directrix.


## disc

A disc is the set of all points on a circle and in the interior of a circle.

## discriminant

The discriminant is the radicand expression in the Quadratic Formula which "discriminates" the number of roots of a quadratic equation.

## Example

The discriminant in the Quadratic Formula is the expression $b^{2}-4 a c$.

## disjoint sets

Two or more sets are disjoint sets if they do not have any common elements.

## Example

Let $N$ represent the set of 9 th grade students. Let $T$ represent the set of 10th grade students. The sets $N$ and $T$ are disjoint sets because the two sets do not have any common elements. Any student can be in one grade only.

## Distance Formula

The Distance Formula can be used to calculate the distance between two points.
The distance between points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ is $d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$.

## Example

To calculate the distance between the points ( $-1,4$ ) and $(2,-5)$, substitute the coordinates into the Distance Formula.
$d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
$d=\sqrt{(2+1)^{2}+(-5-4)^{2}}$
$d=\sqrt{3^{2}+(-9)^{2}}$
$d=\sqrt{9+81}$
$d=\sqrt{90}$
$d \approx 9.49$
So, the distance between the points $(-1,4)$ and
$(2,-5)$ is approximately 9.49 units.

## draw

To draw is to create a geometric figure using tools such as a ruler, straightedge, compass, or protractor. A drawing is more accurate than a sketch.

## E

## element

A member of a set is called an element of that set.

## Example

Set $B$ contains the elements $a, b$, and $c$.
$B=\{a, b, c\}$

## endpoint of a ray

An endpoint of a ray is a point at which a ray begins.

## Example

Point $C$ is the endpoint of ray $C D$.


## endpoints of a line segment

An endpoint of a line segment is a point at which a segment begins or ends.

## Examples

Points $A$ and $B$ are endpoints of segment $A B$.


## Euclidean geometry

Euclidean geometry is a complete system of geometry developed from the work of the Greek mathematician Euclid. He used a small number of undefined terms and postulates to systematically prove many theorems.
Euclid's first five postulates are:

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. If two lines are drawn that intersect a third line in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. (This postulate is equivalent to what is known as the parallel postulate.)

## Example



Euclidean geometry

## event

An event is an outcome or a set of outcomes in a sample space.

## Example

A number cube contains the numbers 1 through 6. Rolling a 6 is one event. Rolling an even number is another event.

## expected value

The expected value is the average value when the number of trials in a probability experiment is large.

## experimental probability

Experimental probability is the ratio of the number of times an event occurs to the total number of trials performed.

## Example

You flip a coin 100 times. Heads comes up 53 times. The experimental probability of getting heads is $\frac{53}{100}$.

## exponentiation

Exponentiation means to raise a quantity to a power.

## exterior angle of a polygon

An exterior angle of a polygon is an angle that is adjacent to an interior angle of a polygon.

## Examples

Angle $J H I$ is an exterior angle of quadrilateral $F G H I$. Angle $E D A$ is an exterior angle of quadrilateral $A B C D$.


## external secant segment

An external secant segment is the portion of each secant segment that lies outside of the circle. It begins at the point at which the two secants intersect and ends at the point where the secant enters the circle.

## Example

Segment $H C$ and segment $P C$ are external secant segments.


## extract the square root

To extract a square root, solve an equation of the form $a^{2}=b$ for $a$.

## Example

To extract the square root for the equation $x^{2}=9$, solve for $x$.
$x^{2}=9$
$x= \pm \sqrt{9}$
$x= \pm 3$

## F

## factor an expression

To factor an expression means to use the Distributive Property in reverse to rewrite the expression as a product of factors.

## Example

The expression $2 x+4$ can be factored as $2(x+2)$.

## factored form

A quadratic function written in factored form is in the form $f(x)=a(x-r 1)\left(x-r_{2}\right)$, where $a \neq 0$.

## Example

The function $h(x)=x^{2}-8 x+12$ written in factored form is $(x-6)(x-2)$.

## factorial

The factorial of $n$, written as $n!$, is the product of all non-negative integers less than or equal to $n$.

## Example

$3!=3 \times 2 \times 1=6$

## flow chart proof

A flow chart proof is a proof in which the steps and corresponding reasons are written in boxes. Arrows connect the boxes and indicate how each step and reason is generated from one or more other steps and reasons.

## Example

A flow chart proof is shown for the conditional statement: If $\overline{A B} \cong \overline{C D}$, then $\overline{A C} \cong \overline{B D}$.
Given: $\overline{A B} \cong \overline{C D}$
Prove: $\overline{A C} \cong \overline{B D}$


## focus of a parabola

The focus of a parabola is a point such that all points on the parabola are equidistant from the focus and the directrix.

## Example

The focus of the parabola shown is the point $(0,2)$. The directrix of the parabola shown is the line $y=-2$. All points on the parabola are equidistant from the focus and the directrix.


## frequency table

A frequency table shows the frequency of an item, number, or event appearing in a sample space.

## Example

The frequency table shows the number of times a sum of two number cubes occurred.

| Sum of Two <br> Number Cubes | Frequency |
| :---: | :---: |
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |
| 5 | 4 |
| 6 | 5 |
| 7 | 6 |
| 8 | 5 |
| 9 | 4 |
| 10 | 3 |
| 11 | 2 |
| 12 | 1 |

## G

## general form of a parabola

The general form of a parabola centered at the origin is an equation of the form $A x^{2}+D y=0$ or $B y^{2}+C x=0$.

## Example

The equation for the parabola shown can be written in general form as $x^{2}-2 y=0$.


## geometric mean

The geometric mean of two positive numbers $a$ and $b$ is the positive number $x$ such that $\frac{a}{x}=\frac{x}{b}$.

## Example

The geometric mean of 3 and 12 is 6 .
$\frac{3}{x}=\frac{x}{12}$
$x^{2}=36$
$x=6$

## geometric probability

Geometric probability is probability that involves a geometric measure, such as length, area, volume, and so on.

## Example

A dartboard has the size and shape shown. The gray shaded area represents a scoring section of the dartboard. Calculate the probability that a dart that lands on a random part of the target will land in a gray scoring section.


Calculate the area of the dartboard: $20(20)=400$ in. $^{2}$
There are 4 gray scoring squares with 8 -in. sides and a gray scoring square with $20-8-8=4$-in. sides. Calculate the area of the gray scoring sections: $4(8)(8)+4(4)=272$ in. $^{2}$
Calculate the probability that a dart will hit a gray scoring section: $\frac{272}{400}=0.68=68 \%$.

## great circle of a sphere

The great circle of a sphere is a cross section of a sphere when a plane passes through the center of the sphere.

## Example

## greatest integer function <br> (floor function)

The greatest integer function, also known as a floor function, is defined as the greatest integer less than or equal to $x$.

## Example

The greatest integer function is defined as $G(x)=\lfloor x \mid$. If $x=3.75$ then $G(x)=3$.

## H

## half-closed (half-open) interval

A half-closed or half-open interval $(a, b]$ describes the set of all numbers between $a$ and $b$, including $b$ but not including $a$. The half-closed interval $[a, b)$ describes the set of all numbers between $a$ and $b$, including $a$ but not including $b$.

## Example

The interval $(3,7]$ is the set of all numbers greater than 3 and less than or equal to 7 .
The interval $[3,7)$ is the set of all numbers greater than or equal to 3 and less than 7 .

## hemisphere

A hemisphere is half of a sphere bounded by a great circle.

## Example

A hemisphere is shown.


## hypothesis

A hypothesis is the "if" part of an "if-then" statement.

## Example

In the statement, "If the last digit of a number is a 5, then the number is divisible by 5 ," the hypothesis is "If the last digit of a number is a 5 ."

## I

## image

An image is a new figure formed by a transformation.

## Example

The figure on the right is the image that has been created by translating the original figure 3 units to the right horizontally.


## the imaginary number $i$

The number $i$ is a number such that $i^{2}=-1$.

## imaginary numbers

The set of imaginary numbers is the set of all numbers written in the form $a+b i$, where $a$ and $b$ are real numbers and $b$ is not equal to 0 .

## imaginary part of a complex number

In a complex number of the form $a+b i$, the term $b i$ is called the imaginary part of a complex number.

## Example

The imaginary part of the complex number $3+2 i$ is $2 i$.

## imaginary roots/imaginary zeros

Imaginary roots are imaginary solutions to equations.

## Example

The quadratic equation $x^{2}-2 x+2$ has two imaginary roots: $1+i$ and $1-i$.

## incenter

The incenter of a triangle is the point at which the angle bisectors of the triangle intersect.

## Example

Point $X$ is the incenter of triangle $A B C$.


## included angle

An included angle is an angle formed by two consecutive sides of a figure.

## Example

In triangle $A B C$, angle $A$ is the included angle formed by consecutive sides $\overline{A B}$ and $\overline{A C}$.


## included side

An included side is a line segment between two consecutive angles of a figure.

## Example

In triangle $A B C, \overline{A B}$ is the included side formed by consecutive angles $A$ and $B$.


## independent events

Independent events are events for which the occurrence of one event has no impact on the occurrence of the other event.

## Example

You randomly choose a yellow marble, replace the marble in the jar, and then randomly choose a yellow marble again. The events of randomly choosing a yellow marble first and randomly choosing a yellow marble second are independent events because the 1st yellow marble was replaced in the jar.

## indirect measurement

Indirect measurement is a technique that uses proportions to determine a measurement when direct measurement is not possible.

## Example

You can use a proportion to solve for the height $x$ of the flagpole.


$$
\begin{aligned}
\frac{x}{5.5} & =\frac{19+11}{11} \\
\frac{x}{5.5} & =\frac{30}{11} \\
11 x & =165 \\
x & =15
\end{aligned}
$$

The flagpole is 15 feet tall.

## indirect proof or proof by contradiction

An indirect proof, or proof by contradiction, uses the contrapositive. By proving that the contrapositive is true, you prove that the statement is true.

## Example

Given: Triangle $D E F$
Prove: A triangle cannot have more than one obtuse angle.
Given $\triangle D E F$, assume that $\triangle D E F$ has two obtuse angles. So, assume $m \angle D=91^{\circ}$ and $m \angle E=91^{\circ}$. By the Triangle Sum Theorem, $m \angle D+m \angle E+m \angle F=$ $180^{\circ}$. By substitution, $91^{\circ}+91^{\circ}+m \angle F=180^{\circ}$, and by subtraction, $m \angle F=-2^{\circ}$. But, it is not possible for a triangle to have a negative angle, so this is a contradiction. This proves that a triangle cannot have more than one obtuse angle.

## induction

induction is reasoning that involves using specific examples to make a conclusion.

## Example

Sandy draws several triangles, measures the interior angles, and calculates the sum of the measures of the three interior angles. She concludes that the sum of the measures of the three interior angles of a triangle is $180^{\circ}$. Sandy reached the conclusion using induction.

## inscribed angle

An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle.

## Example

Angle $B A C$ is an inscribed angle. The vertex of angle $B A C$ is on the circle and the sides of angle BAC contain the chords $\overline{A B}$ and $\overline{A C}$.


## inscribed polygon

An inscribed polygon is a polygon drawn inside a circle such that each vertex of the polygon is on the circle.

## Example

Quadrilateral $K L M N$ is inscribed in circle $J$.


## integers

The set of integers consists of the set of whole numbers and their opposites.

## Example

The numbers $-12,0$, and 30 are integers.

## intercepted arc

An intercepted arc is formed by the intersections of the sides of an inscribed angle with a circle.

## Example

$\overline{P R}$ is an intercepted arc of inscribed angle $P S R$.


## interior angle of a polygon

An interior angle of a polygon is an angle which is formed by consecutive sides of the polygon or shape.

## Example

The interior angles of $\triangle A B C$ are $\angle A B C, \angle B C A$, and $\angle C A B$.


## intersecting sets

Two or more sets are intersecting sets if they have common elements.

## Example

Let $V$ represent the set of students who are on the girls' volleyball team. Let $M$ represent the set of students who are in the math club. Julia is on the volleyball team and belongs to the math club. The sets $V$ and $M$ are intersecting sets because the two sets have at least one common element, Julia.

## interval

An interval is defined as the set of real numbers between two given numbers.

## Example

The interval $(3,7)$ is the set of all numbers between 3 and 7 , not including 3 or 7 .

## inverse

To state the inverse of a conditional statement, negate both the hypothesis and the conclusion.

Conditional Statement: If $p$, then $q$. Inverse: If not $p$, then not $q$.

## Example

Conditional Statement: If a triangle is equilateral, then it is isosceles.
Inverse: If a triangle is not equilateral, then it is not isosceles.

## inverse cosine

The inverse cosine, or arc cosine, of $x$ is the measure of an acute angle whose cosine is $x$.

## Example

In right triangle $A B C$, if $\cos A=x$, then $\cos ^{-1} x=m \angle A$.


## inverse function

An inverse function takes the output value, performs some operation(s) on this value, and arrives back at the original function's input value.

## Example

The inverse of the function $y=2 x$ is the function $x=2 y$, or $y=\frac{x}{2}$.

## inverse operation

"Undoing," working backward, or retracing steps to return to an original value or position is referred to as using the inverse operation.

## Example

The operations of addition and subtraction are inverse operations.

## inverse sine

The inverse sine, or arc sine, of $x$ is the measure of an acute angle whose sine is $x$.

## Example

In right triangle $A B C$, if $\sin A=x$, then $\sin ^{-1} x=m \angle A$.


## inverse tangent

The inverse tangent (or arc tangent) of $x$ is the measure of an acute angle whose tangent is $x$.

## Example

In right triangle $A B C$, if $\tan A=x$, then $\tan ^{-1} x=m \angle A$.


## irrational numbers

The set of irrational numbers consists of all numbers that cannot be written as $\frac{a}{b}$ where $a$ and $b$ are integers.

## Example

The number $\pi$ is an irrational number.

## isometric paper

Isometric paper is often used by artists and engineers to create three-dimensional views of objects in two dimensions.

## Example

The rectangular prism is shown on isometric paper.


## isosceles trapezoid

An isosceles trapezoid is a trapezoid whose nonparallel sides are congruent.

## Example

In trapezoid $J K L M$, side $\overline{K L}$ is parallel to side $\overline{J M}$, and the length of side $\overline{J K}$ is equal to the length of side $\overline{L M}$, so trapezoid JKLM is an isosceles trapezoid.


## I

## Law of Cosines

The Law of Cosines, or

$$
\begin{aligned}
& a^{2}=c^{2}+b^{2}-2 b c \cdot \cos A \\
& b^{2}=a^{2}+c^{2}-2 a c \cdot \cos B \\
& c^{2}=a^{2}+b^{2}-2 a b \cdot \cos C
\end{aligned}
$$

can be used to determine the unknown lengths of sides or the unknown measures of angles in any triangle.


## Example

In triangle $A B C$, the measure of angle $A$ is $65^{\circ}$, the length of side $b$ is 4.4301 feet, and the length of side $c$ is 7.6063 feet. Use the Law of Cosines to calculate the length of side $a$.
$a^{2}=4.4301^{2}+7.6063^{2}-2(4.4301)(7.6063) \cos 65^{\circ}$
The length of side $a$ is 7 feet.

## Law of Sines

The Law of Sines, or $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$, can be used to determine the unknown side lengths or the unknown angle measures in any triangle.

## Example



In triangle $A B C$, the measure of angle $A$ is $65^{\circ}$, the measure of angle $B$ is $80^{\circ}$, and the length of side $a$ is 7 feet. Use the Law of Sines to calculate the length of side $b$.
$\frac{7}{\sin 65^{\circ}}=\frac{b}{\sin 80^{\circ}}$
The length of side $b$ is 7.6063 feet.

## leading coefficient

The leading coefficient of a function is the numerical coefficient of the term with the greatest power.

## Example

In the function $h(x)=-7 x^{2}+x+25$, the value -7 is the leading coefficient.

## least integer function (ceiling function)

The least integer function, also known as the ceiling function, is defined as the least integer greater than or equal to $x$.

## Example

The least integer function is defined as $L(x)=\lceil x\rceil$. If $x=3.75$ then $L(x)=4$.

## line

A line is made up of an infinite number of points that extend infinitely in two opposite directions. A line is straight and has only one dimension.

## Example

The line below can be called line $k$ or line $A B$.

## line segment

A line segment is a portion of a line that includes two points and all of the collinear points between the two points.

## Example

The line segment shown is named $\overline{A B}$ or $\overline{B A}$.


## linear pair

A linear pair of angles are two adjacent angles that have noncommon sides that form a line.

## Example

The diagram shown has four pairs of angles that form a linear pair.

- Angles 1 and 2 form a linear pair.
- Angles 2 and 3 form a linear pair.
- Angles 3 and 4 form a linear pair.
- Angles 4 and 1 form a linear pair.



## linear velocity

Linear velocity is a type of circular velocity described as an amount of distance over a specified amount of time. Linear velocity can be expressed as $v=\frac{s}{t}$, where $v=$ velocity, $s=\operatorname{arc}$ length, and $t=$ time.

## locus of points

A locus of points is a set of points that satisfy one or more conditions.

## Example

A circle is defined as a locus of points that are a fixed distance, called the radius, from a given point, called the center.


## M

## major arc

Two points on a circle determine a major arc and a minor arc. The arc with the greater measure is the major arc. The other arc is the minor arc.

## Example

Circle $Q$ is divided by points $A$ and $B$ into two arcs, $\operatorname{arc} A C B$ and $\operatorname{arc} A B$. Arc $A C B$ has the greater measure, so it is the major arc. Arc $A B$ has the lesser measure, so it is the minor arc.


## median

The median of a triangle is a line segment drawn from a vertex to the midpoint of the opposite side.

## Example

The 3 medians are drawn on the triangle shown.


## midpoint

The midpoint of a line segment is the point that divides the line segment into two congruent segments.

## Example

Because point $B$ is the midpoint of $\overline{A C}, \overline{A B} \cong \overline{B C}$.


## Midpoint Formula

The Midpoint Formula can be used to calculate the midpoint between two points. The midpoint between $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ is $\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$.

## Example

To calculate the midpoint between the points ( $-1,4$ ) and $(2,-5)$, substitute the coordinates into the Midpoint Formula.

$$
\begin{aligned}
\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right) & =\left(\frac{-1+2}{2}, \frac{4-5}{2}\right) \\
& =\left(\frac{1}{2}, \frac{-1}{2}\right)
\end{aligned}
$$

So, the midpoint between the points $(-1,4)$ and $(2,-5)$ is $\left(\frac{1}{2},-\frac{1}{2}\right)$.

## midsegment of a trapezoid

The midsegment of a trapezoid is a line segment formed by connecting the midpoints of the legs of the trapezoid.

## Example

Segment $X Y$ is the midsegment of trapezoid $A B C D$.


## midsegment of a triangle

A midsegment of a triangle is a line segment formed by connecting the midpoints of two sides of a triangle.

## Example

Segment $A B$ is a midsegment.


## minor arc

Two points on a circle determine a minor arc and a major arc. The arc with the lesser measure is the minor arc. The other arc is the major arc.

## Example

Circle $Q$ is divided by points $A$ and $B$ into two arcs, $\operatorname{arc} A C B$ and arc $A B$. Arc $A B$ has the lesser measure, so it is the minor arc. Arc $A C B$ has the greater measure, so it is the major arc.


## monomial

Polynomials with only one term are monomials.

## Example

The expressions $5 x, 7,-2 x y$, and $13 x^{3}$ are monomials.

## N

## natural numbers

The set of natural numbers consists of the numbers that you use to count objects.

## Example

The numbers 1, 2, 3, 4, $\ldots$ are natural numbers.

## negative square root

A square root that is negative.

## Example

The negative square root of 9 is -3 .

## non-uniform probability model

When all probabilities in a probability model are not equivalent to each other, it is called a non-uniform probability model.

## Example

Spinning the spinner shown represents a non-uniform probability model because the probability of landing on a shaded space is not equal to the probability of landing on a non-shaded space.


## 0

## oblique cylinder

When a circle is translated through space in a direction that is not perpendicular to the plane containing the circle, the solid formed is an oblique cylinder.

## Example

The prism shown is an oblique cylinder.


## oblique rectangular prism

When a rectangle is translated through space in a direction that is not perpendicular to the plane containing the rectangle, the solid formed is an oblique rectangular prism.

## Example

The prism shown is an oblique rectangular prism.


## oblique triangular prism

When a triangle is translated through space in a direction that is not perpendicular to the plane containing the triangle, the solid formed is an oblique triangular prism.

## Example

The prism shown is an oblique triangular prism.


## one-to-one function

A function is a one-to-one function if both the function and its inverse are functions.

## Example

The equation $y=x^{3}$ is a one-to-one function because its inverse, $\sqrt[3]{x}=y$, is a function. The equation $y=x^{2}$ is not a one-to-one function because its inverse, $\pm \sqrt{\mathrm{X}}=y$, is not a function.

## open interval

An open interval $(a, b)$ describes the set of all numbers between $a$ and $b$, but not including $a$ or $b$.

## Example

The interval $(3,7)$ is the set of all numbers greater than 3 and less than 7.

## opposite side

The opposite side of a triangle is the side opposite the reference angle.

## Example



## organized list

An organized list is a visual model for determining the sample space of events.

## Example

The sample space for flipping a coin 3 times can be represented as an organized list.

| HHH | THH |
| :--- | :--- |
| HHT | THT |
| HTH | TTH |
| HTT | TTT |

## orthocenter

The orthocenter of a triangle is the point at which the altitudes of the triangle intersect.

## Example

Point $X$ is the orthocenter of triangle $A B C$.


## outcome

An outcome is the result of a single trial of an experiment.

## Example

Flipping a coin has two outcomes: heads or tails.

## P

## parabola

The shape that a quadratic function forms when graphed is called a parabola. A parabola is the set of all points in a plane that are equidistant from a fixed point called the focus and a fixed line called the directrix.

## Example

The focus of the parabola shown is the point $(0,2)$. The directrix of the parabola shown is the line $y=-2$. All points on the parabola are equidistant from the focus and the directrix.


## paragraph proof

A paragraph proof is a proof that is written in paragraph form. Each sentence includes mathematical statements that are organized in logical steps with reasons.

## Example

The proof shown is a paragraph proof that vertical angles 1 and 3 are congruent.
Angle 1 and angle 3 are vertical angles. By the definition of linear pair, angle 1 and angle 2 form a linear pair. Angle 2 and angle 3 also form a linear pair. By the Linear Pair Postulate, angle 1 and angle 2 are supplementary. Angle 2 and angle 3 are also supplementary. Angle 1 is congruent to angle 3 by the Congruent Supplements Theorem.


## perfect square trinomial

A perfect square trinomial is an expression in the form $a^{2}+2 a b+b^{2}$ or in the form $a^{2}-2 a b+b^{2}$.

## Example

The trinomial $x^{2}+6 x+9$ is a perfect square trinomial because it can be written as $x^{2}+2(3) x+3^{2}$.

## permutation

A permutation is an ordered arrangement of items without repetition.

## Example

The permutations of the letters $A, B$, and $C$ are:

| $A B C$ | $A C B$ |
| :--- | :--- |
| $B A C$ | $B C A$ |
| $C A B$ | $C B A$ |

## perpendicular bisector

A perpendicular bisector is a line, line segment, or ray that intersects the midpoint of a line segment at a 90-degree angle.

## Example

Line $k$ is the perpendicular bisector of $\overline{A B}$. It is perpendicular to $\overline{A B}$, and intersects $\overline{A B}$ at midpoint $M$ so that $A M=M B$.


## plane

A plane is a flat surface with infinite length and width, but no depth. A plane extends infinitely in all directions.

## Example

Plane $A$ is shown.

## point

A point has no dimension, but can be visualized as a specific position in space, and is usually represented by a small dot.

## Example

point $A$ is shown.

## ${ }^{\bullet}{ }_{A}$

## point of concurrency

A point of concurrency is the point at which three or more lines intersect.

## Example

Point $X$ is the point of concurrency for lines $\ell, m$, and $n$.


## point of tangency

A tangent to a circle is a line that intersects the circle at exactly one point, called the point of tangency.

## Example

Line $R Q$ is tangent to circle $P$. Point $Q$ is the point of tangency.


## point-slope form

The point-slope form of a linear equation that passes through the point $\left(x_{1}, y_{1}\right)$ and has slope $m$ is $y-y_{1}=m\left(x-x_{1}\right)$.

## Example

A line passing through the point $(1,2)$ with a slope of $\frac{1}{2}$ can be written in point-slope form as
$y-2=\frac{1}{2}(x+1)$.

## polynomial

A polynomial is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients.

## Example

The expression $3 x^{3}+5 x-6 x+1$ is a polynomial.

## positive square root

A square root that is positive.

## Example

The positive square root of 9 is 3 .

## postulate

A postulate is a statement that is accepted to be true without proof.

## Example

The following statement is a postulate: A straight line may be drawn between any two points.

## pre-image

A pre-image is the figure that is being transformed.

## Example

The figure on the right is the image that has been formed by translating the pre-image 3 units to the right horizontally.


## principal square root

A positive square root of a number.

## Example

The principal square root of 9 is 3 .

## principal square root of a negative number

For any positive real number $n$, the principal square root of a negative number, $-n$, is defined by $\sqrt{-n}=i \sqrt{n}$.

## Example

The principal square root of -5 is $\sqrt{-5}=i \sqrt{5}$.

## probability

The probability of an event is the ratio of the number of desired outcomes to the total number of possible outcomes, $P(A)=\frac{\text { desired outcomes }}{\text { possible outcomes }}$.

## Example

When flipping a coin, there are 2 possible outcomes: heads or tails. The probability of flipping a heads is $\frac{1}{2}$.

## probability model

A probability model lists the possible outcomes and the probability for each outcome. In a probability model, the sum of the probabilities must equal 1.

## Example

The table shows a probability model for flipping a fair coin once.

| Outcomes | Heads $(H)$ | Tails $(T)$ |
| :---: | :---: | :---: |
| Probability | $\frac{1}{2}$ | $\frac{1}{2}$ |

## propositional form

When a conditional statement is written using the propositional variables $p$ and $q$, the statement is said to be written in propositional form.

## Example

Propositional form:
"If $p$, then $q$."
$p \rightarrow q$

## propositional variables

When a conditional statement is written in propositional form as "If $p$, then $q$," the variables $p$ and $q$ are called propositional variables.

## pure imaginary number

A pure imaginary number is a number of the form bi, where $b$ is not equal to 0 .

## Example

The imaginary numbers $24 i$ and $15 i$ are pure imaginary numbers.

## Q

## Quadratic Formula

The Quadratic Formula is $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$.

## quadratic regression

A quadratic regression is a mathematical method to determine the equation of a "parabola of best fit" for a data set.

## Example

The graph of the quadratic regression for these data is shown.


## R

## radian

One radian is defined as the measure of a central angle whose arc length is the same as the radius of the circle.

## radical expression

A radical expression is an expression that involves a radical symbol $(\sqrt{ })$.

## radicand

The value that is inside a radical is called the radicand.

## Example

In the radical expression $\sqrt{25}$, the number 25 is the radicand.

## radius

The radius of a circle is a line segment with one endpoint on the circle and one endpoint at the center.

## Example

In circle $O, \overline{O A}$ is a radius.


## radius of a sphere

The radius of a sphere is a line segment with one endpoint on the sphere and one endpoint at the center.

Example


## rational numbers

The set of rational numbers consists of all numbers that can be written as $\frac{a}{b}$ where $a$ and $b$ are integers, but $b$ is not equal to 0 .

## Example

The number 0.5 is a rational number because it can be written as the fraction $\frac{1}{2}$.

## rationalizing the denominator

Rationalizing the denominator is the process of eliminating a radical from the denominator of an expression. To rationalize the denominator, multiply by a form of one so that the radicand of the radical in the denominator is a perfect square.

## Example

Rationalize the denominator of the expression $\frac{5}{\sqrt{3}}$.

$$
\begin{aligned}
\frac{5}{\sqrt{3}} & =\frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\
& =\frac{5 \sqrt{3}}{\sqrt{9}} \\
& =\frac{5 \sqrt{3}}{3}
\end{aligned}
$$

## ray

A ray is a portion of a line that begins with a single point and extends infinitely in one direction.

## Example

The ray shown is ray $A B$.


## real numbers

The set of real numbers consists of the set of rational numbers and the set of irrational numbers.

## Examples

The numbers $-3,11.4, \frac{1}{2}$, and $\sqrt{5}$ are real numbers.

## real part of a complex number

In a complex number of the form $a+b i$, the term $a$ is called the real part of a complex number.

## Example

The real part of the complex number $3+2 i$ is 3 .

## reference angle

A reference angle is the angle of the right triangle being considered. The opposite side and adjacent side are named based on the reference angle.

## Example



## Reflexive Property

The reflexive property states that $a=a$.

## Example

The statement $2=2$ is an example of the reflexive property.

## relative frequency

A relative frequency is the ratio or percent of occurrences within a category to the total of the category.

## Example

John surveys 100 students in his school about their favorite school subject. Of the 100 students, 37 chose math as their favorite subject. The relative frequency of students show selected math as their favorite subject is $\frac{37}{100}$, or $37 \%$.

## remote interior angles of a triangle

The remote interior angles of a triangle are the two angles that are not adjacent to the specified exterior angles.

## Example

The remote interior angles with respect to exterior angles 4 are angles 1 and 2 .


## restrict the domain

To restrict the domain of a function means to define a new domain for the function that is a subset of the original domain.

## right cylinder

A disc translated through space in a direction perpendicular to the plane containing the disc forms a right cylinder.

## Example



## right rectangular prism

A rectangle translated through space in a direction perpendicular to the plane containing the rectangle forms a right rectangular prism.

## Example



## right triangular prism

A triangle translated through space in a direction perpendicular to the plane containing the triangle forms

## Example



## rigid motion

A rigid motion is a transformation of points in space. Translations, reflections, and rotations are examples of rigid motion.

## roots

The roots of a quadratic equation indicate where the graph of the equation crosses the $x$-axis.

## Example

The roots of the quadratic equation $x^{2}-4 x=-3$ are $x=3$ and $x=1$.

## Rule of Compound Probability involving "and"

The Rule of Compound Probability involving "and" states: "If Event $A$ and Event $B$ are independent, then the probability that Event $A$ happens and Event $B$ happens is the product of the probability that Event $A$ happens and the probability that Event $B$ happens, given that Event $A$ has happened."

$$
P(A \text { and } B)=P(A) \cdot P(B)
$$

## Example

You flip a coin two times. Calculate the probability of flipping a heads on the first flip and flipping a heads on the second flip.
Let $A$ represent the event of flipping a heads on the first flip. Let $B$ represent the event of flipping a heads on the second flip.
$P(A$ and $B)=P(A) \cdot P(B)$
$P(A$ and $B)=\frac{1}{2} \cdot \frac{1}{2}$
$P(A$ or $B)=\frac{1}{4}$
So, the probability of flipping a heads on the first flip and flipping a heads on the second flip is $\frac{1}{4}$.


## sample space

A list of all possible outcomes of an experiment is called a sample space.

## Example

Flipping a coin two times consists of four outcomes: HH, HT, TH, and TT.

## secant (sec)

The secant (sec) of an acute angle in a right triangle is the ratio of the length of the hypotenuse to the length of the side adjacent to the angle.

## Example

In triangle $A B C$, the secant of angle $A$ is:
$\sec A=\frac{\text { length of hypotenuse }}{\text { length of side adjacent to } \angle A}=\frac{A B}{A C}$
The expression "sec $A$ " means "the secant of angle $A$."


## secant of a circle

A secant of a circle is a line that intersects the circle at two points.

## Example

The line intersecting the circle through points $A$ and $B$ is a secant.


## secant segment

A secant segment is formed when two secants intersect outside of a circle. A secant segment begins at the point at which the two secants intersect, continues into the circle, and ends at the point at which the secant exits the circle.

## Example

Segment GC and segment $N C$ are secant segments.


## second differences

Second differences are the differences between consecutive values of the first differences.

## Example

| $\boldsymbol{x}$ | $y$ | First Differences | Second Differences |
| :---: | :---: | :---: | :---: |
| -3 |  |  |  |
|  | -5 | 5 |  |
| -2 |  |  | -2 |
|  |  | 3 |  |
| -1 | 3 | 1 | -2 |
|  |  |  |  |
| 0 | 4 | -1 | -2 |
| 1 |  |  | -2 |
|  | 3 | -3 |  |
| 2 | 0 |  | -2 |
|  |  | -5 |  |
| 3 | -5 |  |  |

## sector of a circle

A sector of a circle is a region of the circle bounded by two radii and the included arc.

## Example

In circle $Y$, arc $X Z$, radius $X Y$, and radius $Y Z$ form a sector.


## segment bisector

A segment bisector is a line, line segment, or ray that intersects a line segment so that the line segment is divided into two segments of equal length.

## Example

Line $k$ is a segment bisector of segment $A C$. The lengths of segments $A B$ and $B C$ are equal.


## segment of a circle

A segment of a circle is a region bounded by a chord and the included arc.

## Example

In circle $A$, chord $\overline{B C}$ and arc $B C$ are the boundaries of a segment of the circle.


## segments of a chord

Segments of a chord are the segments formed on a chord if two chords of a circle intersect.

## Example

The segments of chord $\overline{H D}$ are $\overline{E H}$ and $\overline{E D}$. The segments of chord $\overline{R C}$ are $\overline{E R}$ and $\overline{E C}$.


## semicircle

A semicircle is an arc whose endpoints form the endpoints of a diameter of the circle.

## Example

Arc $X Y Z$ and arc $Z W X$ are semicircles of circle $P$.


## set

A set is a collection of items. If $x$ is a member of $\operatorname{set} B$, then $x$ is an element of set $B$.

## Example

Let $E$ represent the set of even whole numbers.
$E=\{2,4,6,8, \ldots\}$

## similar triangles

Similar triangles are triangles that have all pairs of corresponding angles congruent and all corresponding sides are proportional.

## Example

Triangle $A B C$ is similar to triangle $D E F$.


## simulation

A simulation is an experiment that models a real-life situation.

## Example

You can simulate the selection of raffle numbers by using the random number generator on a graphing calculator.

## sine (sin)

The sine (sin) of an acute angle in a right triangle is the ratio of the length of the side opposite the angle to the length of the hypotenuse.

## Example

In triangle $A B C$, the sine of angle $A$ is:
$\sin A=\frac{\text { length of side opposite } \angle A}{\text { length of hypotenuse }}=\frac{B C}{A B}$
The expression "sin $A$ " means "the sine of angle $A$."


## sketch

To sketch is to create a geometric figure without using tools such as a ruler, straightedge, compass, or protractor. A drawing is more accurate than a sketch.

## skew lines

Skew lines are two lines that do not intersect and are not parallel. Skew lines do not lie in the same plane.

## Example

Line $m$ and line $p$ are skew lines.


## sphere

A sphere is the set of all points in space that are a given distance from a fixed point called the center of the sphere.

## Example

A sphere is shown.


## square root

A number $b$ is a square root of $a$ if $b^{2}=a$.

## Example

The number 3 is a square root of 9 because $3^{2}=9$.

## standard form (general form) of a quadratic function

A quadratic function written in the form $f(x)=a x^{2}+b x+c$, where $a \neq 0$, is in standard form, or general form.

## Example

The function $f(x)=-5 x^{2}-10 x+1$ is written in standard form.

## standard form of a parabola

The standard form of a parabola centered at the origin is an equation of the form $x^{2}=4 p y$ or $y^{2}=4 p x$, where $p$ represents the distance from the vertex to the focus.

## Example

The equation for the parabola shown can be written in standard form as $x^{2}=2 y$.


## step function

A step function is a piecewise function whose pieces are disjoint constant functions.

## Example



## straightedge

A straightedge is a ruler with no numbers.

## Substitution Property of Equality

The Substitution Property of Equality states: "If $a$ and $b$ are real numbers and $a=b$, then $a$ can be substituted for $b$."

## Example

If $A B=12 \mathrm{ft}$ and $C D=12 \mathrm{ft}$, then $A B=C D$.

## Subtraction Property of Equality

The Subtraction Property of Equality states: "If $a=b$, then $a-c=b-c$."

## Example

If $x+5=7$, then $x+5-5=7-5$, or $x=2$ is an example of the subtraction property of equality.

## sum of two cubes

The sum of two cubes is an expression in the form $a^{3}+b^{3}$ that can be factored as $(a+b)\left(a^{2}-a b+b^{2}\right)$.

## Example

The expression $x^{3}+8$ is a sum of two cubes because it can be written in the form $x^{3}+2^{3}$. The expression can be factored as $(x+2)\left(x^{2}-2 x+4\right)$.

## supplementary angles

Two angles are supplementary if the sum of their measures is $180^{\circ}$.

## Example

Angle 1 and angle 2 are supplementary angles.
If $m \angle 1=75^{\circ}$, then $m \angle 2=180^{\circ}-75^{\circ}=105^{\circ}$.


## T

## tangent (tan)

The tangent (tan) of an acute angle in a right triangle is the ratio of the length of the side opposite the angle to the length of the side adjacent to the angle.

## Example

In triangle $A B C$, the tangent of angle $A$ is:
$\tan A=\frac{\text { length of side opposite } \angle A}{\text { length of side adjacent to } \angle A}=\frac{B C}{A C}$
The expression "tan $A$ " means "the tangent of angle $A$."


## tangent of a circle

A tangent of a circle is a line that intersects the circle at exactly one point, called the point of tangency.

## Example

Line $R Q$ is tangent to circle $P$.


## tangent segment

A tangent segment is a line segment formed by connecting a point outside of the circle to a point of tangency.

## Example

Line segment $A B$ and line segment $A C$ are tangent segments.


## term

Within a polynomial, each product is a term.

## Example

The polynomial $2 x+3 y+5$ has three terms: $2 x, 3 y$, and 5.

## theorem

A theorem is a statement that has been proven to be true.

## Example

The Pythagorean Theorem states that if a right triangle has legs of lengths $a$ and $b$ and hypotenuse of length $c$, then $a^{2}+b^{2}=c^{2}$.

## theoretical probability

Theoretical probability is the mathematical calculation that an event will happen in theory.

## Example

The theoretical probability of rolling a 1 on a number cube is $\frac{1}{6}$.

## transformation

A transformation is an operation that maps, or moves, a figure, called the preimage, to form a new figure called the image. Three types of transformations are reflections, rotations, and translations.

## Example



## Transitive Property of Equality

The Transitive Property of Equality states: "If $a=b$ and $b=c$, then $a=c$."

## Example

If $x=y$ and $y=2$, then $x=2$ is an example of the Transitive Property of Equality.

## translation

A translation is a transformation in which a figure is shifted so that each point of the figure moves the same distance in the same direction. The shift can be in a horizontal direction, a vertical direction, or both.

## Example

The top trapezoid is a vertical translation of the bottom trapezoid by 5 units.


## tree diagram

A tree diagram is a diagram that illustrates sequentially the possible outcomes of a given situation.

## Example

## trinomial

Polynomials with exactly three terms are trinomials.

## Example

The polynomial $5 x^{2}-6 x+9$ is a trinomial.

## truth table

A truth table is a table that summarizes all possible truth values for a conditional statement $p \rightarrow q$. The first two columns of a truth table represent all possible truth values for the propositional variables $p$ and $q$. The last column represents the truth value of the conditional statement $p \rightarrow q$.

## Example

The truth value of the conditional statement $p \rightarrow q$ is determined by the truth value of $p$ and the truth value of $q$.

- If $p$ is true and $q$ is true, then $p \rightarrow q$ is true.
- If $p$ is true and $q$ is false, then $p \rightarrow q$ is false.
- If $p$ is false and $q$ is true, then $p \rightarrow q$ is true.
- If $p$ is false and $q$ is false, then $p \rightarrow q$ is true.

| $p$ | $q$ | $p \rightarrow q$ |
| :---: | :---: | :---: |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

## truth value

The truth value of a conditional statement is whether the statement is true or false. If a conditional statement could be true, then the truth value of the statement is considered true. The truth value of a conditional statement is either true or false, but not both.

## Example

The truth value of the conditional statement "If a
quadrilateral is a rectangle, then it is a square" is false.

## two-column proof

A two-column proof is a proof consisting of two columns. In the left column are mathematical statements that are organized in logical steps. In the right column are the reasons for each mathematical statement.

## Example

The proof shown is a two-column proof.
Statements

1. $\angle 1$ and $\angle 3$ are vertical
angles.
2. $\angle 1$ and $\angle 2$ form a
linear pair. $\angle 2$ and $\angle 3$
form a linear pair.
3. $\angle 1$ and $\angle 2$ are
supplementary. $\angle 2$ and
$\angle 3$ are supplementary.
4. $\angle 1 \cong \angle 3$
Statements
5. $\angle 1$ and $\angle 3$ are vertical
angles.
6. $\angle 1$ and $\angle 2$ form a
linear pair. $\angle 2$ and $\angle 3$
form a linear pair.
7. $\angle 1$ and $\angle 2$ are
supplementary. $\angle 2$ and
$\angle 3$ are supplementary.
8. $\angle 1 \cong \angle 3$
Statements
9. $\angle 1$ and $\angle 3$ are vertical
angles.
10. $\angle 1$ and $\angle 2$ form a
linear pair. $\angle 2$ and $\angle 3$
form a linear pair.
11. $\angle 1$ and $\angle 2$ are
supplementary. $\angle 2$ and
$\angle 3$ are supplementary.
12. $\angle 1 \cong \angle 3$
Statements
13. $\angle 1$ and $\angle 3$ are vertical
angles.
14. $\angle 1$ and $\angle 2$ form a
linear pair. $\angle 2$ and $\angle 3$
form a linear pair.
15. $\angle 1$ and $\angle 2$ are
supplementary. $\angle 2$ and
$\angle 3$ are supplementary.
16. $\angle 1 \cong \angle 3$
Statements
17. $\angle 1$ and $\angle 3$ are vertical
angles.
18. $\angle 1$ and $\angle 2$ form a
linear pair. $\angle 2$ and $\angle 3$
form a linear pair.
19. $\angle 1$ and $\angle 2$ are
supplementary. $\angle 2$ and
$\angle 3$ are supplementary.
20. $\angle 1 \cong \angle 3$
Statements
21. $\angle 1$ and $\angle 3$ are vertical
angles.
22. $\angle 1$ and $\angle 2$ form a
linear pair. $\angle 2$ and $\angle 3$
form a linear pair.
23. $\angle 1$ and $\angle 2$ are
supplementary. $\angle 2$ and
$\angle 3$ are supplementary.
24. $\angle 1 \cong \angle 3$
Statements
25. $\angle 1$ and $\angle 3$ are vertical
angles.
26. $\angle 1$ and $\angle 2$ form a
linear pair. $\angle 2$ and $\angle 3$
form a linear pair.
27. $\angle 1$ and $\angle 2$ are
supplementary. $\angle 2$ and
$\angle 3$ are supplementary.
28. $\angle 1 \cong \angle 3$
Statements
29. $\angle 1$ and $\angle 3$ are vertical
angles.
30. $\angle 1$ and $\angle 2$ form a
linear pair. $\angle 2$ and $\angle 3$
form a linear pair.
31. $\angle 1$ and $\angle 2$ are
supplementary. $\angle 2$ and
$\angle 3$ are supplementary.
32. $\angle 1 \cong \angle 3$
Statements
33. $\angle 1$ and $\angle 3$ are vertical
angles.
34. $\angle 1$ and $\angle 2$ form a
linear pair. $\angle 2$ and $\angle 3$
form a linear pair.
35. $\angle 1$ and $\angle 2$ are
supplementary. $\angle 2$ and
$\angle 3$ are supplementary.
36. $\angle 1 \cong \angle 3$

| Statements |
| :--- |
| 1. $\angle 1$ and $\angle 3$ are vertical |
| angles. |
| 2. $\angle 1$ and $\angle 2$ form a |
| linear pair. $\angle 2$ and $\angle 3$ |
| form a linear pair. |
| 3. $\angle 1$ and $\angle 2$ are |
| supplementary. $\angle 2$ and |
| $\angle 3$ are supplementary. |
| 4. $\angle 1 \cong \angle 3$ |

## two-way frequency table (contingency table)

A two-way frequency table, also called a contingency table, shows the number of data points and their frequencies for two variables. One variable is divided into rows, and the other is divided into columns.

## Example

The two-way frequency table shows the hand(s) favored by people who do and do not participate in individual or team sports.


## two-way relative frequency table

A two-way relative frequency table displays the relative frequencies for two categories of data.

## Example

The two-way relative frequency table shows the hand(s) favored by people who do and do not participate in individual or team sports.

|  | Individual | Team | Does Not Play | Total |
| :---: | :---: | :---: | :---: | :---: |
| Left | $\frac{3}{63} \approx 4.8 \%$ | $\frac{13}{63} \approx 20.6 \%$ | $\frac{8}{63} \approx 12.7 \%$ | $\frac{24}{63} \approx 38.1 \%$ |
| Right | $\frac{6}{63} \approx 9.5 \%$ | $\frac{23}{63} \approx 36.5 \%$ | $\frac{4}{63} \approx 6.3 \%$ | $\frac{33}{63} \approx 52.4 \%$ |
| Mixed | $\frac{1}{63} \approx 1.6 \%$ | $\frac{3}{63} \approx 4.8 \%$ | $\frac{2}{63} \approx 3.2 \%$ | $\frac{6}{63} \approx 9.5 \%$ |
| Total | $\frac{10}{63} \approx 15.9 \%$ | $\frac{39}{63} \approx 61.9 \%$ | $\frac{14}{63} \approx 22.2 \%$ | $\frac{63}{63}=100 \%$ |

## two-way table

A two-way table shows the relationship between two data sets, one data set is organized in rows and the other data set is organized in columns.

## Example

The two-way table shows all the possible sums that result from rolling two number cubes once.

## 2nd Number Cube



## U

## uniform probability model

A uniform probability model occurs when all the probabilities in a probability model are equally likely to occur.

## Example

Rolling a number cube represents a uniform probability model because the probability of rolling each number is equal.

## Venn diagram

A Venn diagram uses circles to show how elements among sets of numbers or objects are related.

## vertex angle of an isosceles triangle

The vertex angle of an isosceles triangle is the angle formed by the two congruent legs.

## Example



## vertex form

A quadratic function written in vertex form is in the form $f(x)=a(x-h)^{2}+k$, where $a \neq 0$.

## Example

The quadratic equation $y=2(x-5)^{2}+10$ is written in vertex form. The vertex of the graph is the point $(5,10)$.

Example
Whole numbers 1 -10


## vertex of a parabola

The vertex of a parabola, which lies on the axis of symmetry, is the highest or lowest point on the parabola.

## Example

The vertex of the parabola is the point $(1,-4)$, the minimum point on the parabola.


## vertical angles

Vertical angles are two nonadjacent angles that are formed by two intersecting lines.

## Examples

Angles 1 and 3 are vertical angles.
Angles 2 and 4 are vertical angles.


## vertical dilation

A vertical dilation of a function is a transformation in which the $y$-coordinate of every point on the graph of the function is multiplied by a common factor.

## Example

The coordinate notation $(x, y) \rightarrow(x, a y)$, where $a$ is the dilation factor, indicates a vertical dilation.

## vertical motion model

A vertical motion model is a quadratic equation that models the height of an object at a given time. The equation is of the form $g(t)=-16 t^{2}+v 0 t+h 0$, where $g(t)$ represents the height of the object in feet, $t$ represents the time in seconds that the object has been moving, $v 0$ represents the initial velocity (speed) of the object in feet per second, and $h 0$ represents the initial height of the object in feet.

## Example

A rock is thrown in the air at a velocity of 10 feet per second from a cliff that is 100 feet high. The height of the rock is modeled by the equation $y=-16 t^{2}+10 t+100$.

## W

## whole numbers

The set of whole numbers consists of the set of natural numbers and the number 0 .

## Example

The numbers $0,1,2,3, \ldots$ are whole numbers.

## Z

## Zero Product Property

The Zero Product Property states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero. This is also called the Converse of Multiplication Property of Zero.

## Example

If $(x-2)(x+3)=0$, then $x-2=0$ or $x+3=0$.

## zeros

The $x$-intercepts of a graph of a quadratic function are also called the zeros of the quadratic function.

## Example

The zeros of the quadratic function $f(x)=-2 x^{2}+4 x$ are $(0,0)$ and $(2,0)$.

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